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Description A utility package containing some simple tools to design and generate density functions on bounded regions in space and space-time, and simulate iid data therefrom. See Davies & Hazelton (2010) <doi:10.1002/sim.3995> for example.
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The spagmix Package: Artificial Spatial and Spatiotemporal Densities on Bounded Windows

Description

Provides functions to design synthetic spatial and spatiotemporal densities and relative risk functions based mainly on Gaussian mixture distributions, and simulate iid data therefrom.

Details

Package: spagmix  
Version: 0.3-1  
Date: 2018-09-08  
License: GPL (>= 2)

Appraisal of existing, refined, and new statistical methods for the analysis of spatial and spatiotemporal point pattern data usually involves numeric experimentation. Motivated by relevant problems in nonparametric density estimation (see e.g. Wand & Jones, 1995), spagmix ("spatial Gaussian mixtures") provides some simple utilities for designing heterogeneous density and density-ratio or relative risk (Bithell 1990, 1991; Kelsall & Diggle, 1995) functions in space and space-time (see Fernando & Hazelton, 2014 for the latter). The package is also capable of producing realisations of (possibly inhomogeneous) spatial log-Gaussian Cox process intensities (Møller et al., 1998; see also Davies & Hazelton, 2013).

Additionally, the package contains functions for simulating datasets given these scenarios. For examples of how these kinds of synthetic functions have been used in simulation studies in various publications, see for example Clark & Lawson, 2004; Davies & Hazelton, 2010; Davies, 2013a,b; Davies & Hazelton, 2013; Fernando et al., 2014; Davies et al., 2016; Davies et al., 2018a; and Davies & Lawson, 2018.

We have designed the objects of spagmix to use and be compatible with standard object classes of the spatstat (Baddeley & Turner, 2005; Baddeley et al., 2015) and sparr (Davies et al., 2018b) packages. The content of spagmix can be broken up as follows:

**Artificial polygonal windows**

Some pre-made synthetic spatial windows; these are all single closed polygons as objects of class owin and are lazy-loaded:
**Spatial scenarios**

*sgmix* is used to create spatial (2D) Gaussian mixture distributions on a bounded subset of the plane. *rgmix* also creates 2D Gaussian mixture densities, but does so by stochastic generation of the contributing bumps.

*rrmix* creates Gaussian mixture relative risk scenarios based on a supplied control density (see e.g. Davies & Hazelton, 2010). *lgcpmix* generates a spatial log-Gaussian Cox process intensity in space, given a deterministic intensity function and residual correlation governed by a stochastic realisation of a Gaussian field with a specified covariance structure.

**Spatiotemporal scenarios**

*stgmix* is used to create spatiotemporal (3D) Gaussian mixture densities on a bounded subset of the plane and a single closed interval in time. *stkey* is used to create spatiotemporal densities by pixel-wise interpolation of multiple spatial image ‘keyframes’. *rrstmix* is a spatiotemporal version of *rrmix*, used to create artificial spatiotemporal relative risk functions. Note the control density may be purely spatial, representing a distribution ‘at-risk’ points that does not change over time (Fernando & Hazelton, 2014).

**Data generation**

To generate purely spatial data for a single spatial density, the user is directed to *rpoint* of the spatstat package or *rimpoly* of the sparr package. *rpoispoly* is a wrapper of *rimpoly*, and is used to generate realisations of Poisson point processes in space, given an intensity function. *rrpoint* is a wrapper of *rimpoly*, and is used to generate iid datasets based on a synthetic spatial relative risk surface object. *rstpoint* is a 3D rejection algorithm for sampling iid data from a supplied spatiotemporal density. *rrstpoint* is a wrapper of *rstpoint* to generate iid datasets from a synthetic spatiotemporal relative risk surface object.

**Miscellaneous**

*plotNstim* is an S3 plotting method for spatiotemporal density objects. *stintegral* computes the 3D integral of a spatiotemporal density object. *unifyNowin* is a wrapper for *affine* that transforms any spatial *owin* to fall inside the unit square.

**Dependencies/Imports**

The spagmix package depends upon *spatstat* (Baddeley & Turner, 2005; Baddeley et al., 2015) and imports from RandomFields (Schlather et al., 2015), abind (Plate & Heiberger, 2016), MCMCpack (Martin et al., 2011), sparr (Davies et al., 2018b), and mvtnorm (Genz et al., 2018). We also highly recommend the rgl package (Adler et al., 2018) which can be used to create interactive plots of spatiotemporal data.

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References

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Kelsall, J.E. and Diggle, P.J. (1995), Kernel estimation of relative risk, Bernoulli, 1, 3-16.


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### lgcpmix

Generate a spatial log-Gaussian Cox process intensity

---

**Description**

Generate a realisation of a (possibly inhomogeneous) log-Gaussian Cox process (LGCP) spatial intensity function with an identifiable mean structure.

**Usage**

```
lgcpmix(lambda, covmodel = "exp", covpars = NULL)
```

**Arguments**

- `lambda`: A pixel image giving the deterministic spatial intensity as the mean structure of the process. The generated Gaussian field will match the dimensions, resolution and domain of this object.
- `covmodel`: A character string giving the short name of a spatial covariance model available in the RandomFields package (Schlather et al., 2015). The code searches for a function of this name after adding the prefix "rm", piggybacking on the internals of rLGCP. See ‘Details’.
- `covpars`: A named list of values for the parameters required by the chosen covmodel. This will normally include var and scale. Both variance and scale parameters default to 1 if this argument is left NULL.
Details

This function allows the user to generate a spatial intensity function $\Gamma$ of the form

$$\Gamma(x) = \lambda(x) \exp[Y(x)]$$

for $x \in W$, where $\lambda(x)$ (passed to lambda) is the deterministic spatial intensity over the spatial domain $W$, and $Y(x)$ is a Gaussian random field on $W$. This Gaussian field is defined with a particular spatial covariance function (specified in covmodel) with variance and scale parameters $\sigma^2$ and $\phi$ respectively, as well as any additionally required parameter values (all specified in covpars).

The mean parameter $\mu$ of the Gaussian field $Y$ is internally fixed at $-\sigma^2/2$; negative half the variance. This is for identifiability of the mean structure, forcing $E[Y(x)] = 1$ for all $x \in W$ (see theoretical properties in Møller et al., 1998). This means the deterministic intensity function $\lambda(x)$ is solely responsible for describing fixed heterogeneity in spatial intensity over $W$, with the randomly generated Gaussian field left to describe residual stochastic spatial correlation. This presents a highly flexible class of model, even with stationarity and isotropy of the Gaussian field itself, and is intuitively sensible in a variety of applications. See Diggle et al. (2005) and Davies & Hazelton (2013) for example.

As such, the pixel image supplied to lambda as $\lambda(x)$ must be non-negatively-valued and yield a finite integral. The choice of covariance model and correspondingly required parameters as well as actual simulation of the Gaussian field is deferred to functionality in the RandomFields package; see RMmodel for possible choices. For example, requesting covmodel = "exp" (default) will search for the RandomFields function RMexp and imposes an exponential covariance structure on the generated field whereby $Cov(u) = \sigma^2 \exp(-u/\phi)$ for the Euclidean distance between any two spatial locations $u$.

To generate a subsequent dataset, use e.g. rpoispp or rpoispoly.

Value

A pixel image giving the generated intensity function, comprised of the product of lambda (fixed, and unchanging in repeated calls to this function) and the exponentiated Gaussian field (with expected value 1, this is stochastic and varies in repeated calls).

Author(s)

T.M. Davies, based partially on code written for the rLGC function by A. Jalilian, R. Waagepetersen, A. Baddeley, R. Turner and E. Rubak.

References


Examples

## Homogeneous example ##

# Create constant intensity image integrating to 500

homog <- as.im(as.mask(toywin))
homog <- homog/integral(homog)*500

# Corresponding LGCP realisations using exponential covariance structure
par(mfrow=c(2,2),mar=rep(1.5,4))
for(i in 1:4){
  temp <- lgcpmix(homog,covmod="exp",covpars=list(var=1,scale=0.2))
  plot(temp,main=paste("Realisation",i),log=TRUE)
}

## Inhomogeneous examples ##

# Create deterministic trend

mn <- cbind(c(0.25,0.8),c(0.31,0.82),c(0.43,0.64),c(0.63,0.62),c(0.21,0.26))
v1 <- matrix(c(0.0023,-0.0009,-0.0009,0.002),2)
v2 <- matrix(c(0.0016,0.0015,0.0015,0.004),2)
v3 <- matrix(c(0.0007,0.0004,0.0004,0.0011),2)
v4 <- matrix(c(0.0023,-0.0041,-0.0041,0.0099),2)
v5 <- matrix(c(0.0013,0.0011,0.0011,0.0014),2)
vr <- array(NA,dim=c(2,2,5))
for(i in 1:5) vr[,i] <- get(paste("v",i,sep=""))
intens <- sgmix(mean=mn,vcv=vr,window=toywin,p0=0.1,int=500)

# Two realisations (identical calls to function), exponential covariance structure

r1exp <- lgcpmix(lambda=intens,covmodel="exp",covpars=list(var=2,scale=0.05))
r2exp <- lgcpmix(lambda=intens,covmodel="exp",covpars=list(var=2,scale=0.05))

# Two more realisations, Matern covariance with smoothness 1

r1mat <- lgcpmix(lambda=intens,covmodel="matern",covpars=list(var=2,scale=0.05,nu=1))
r2mat <- lgcpmix(lambda=intens,covmodel="matern",covpars=list(var=2,scale=0.05,nu=1))

# Plot everything, including 'intens' alone (no correlation)
par(mar=rep(2,4))
### Description

plot method for class `stim`.

### Usage

```r
## S3 method for class 'stim'
plot(x, fix.range = FALSE, sleep = 0.2, override.par = TRUE, ...)
```

### Arguments

- **x**
  - An object of class `stim`.

- **fix.range**
  - Logical value indicating whether use the same color scale limits for each plot in the sequence. Ignored if the user supplies a pre-defined `colourmap` to the `col` argument, which is matched to ... and passed to `plot.im`.

- **sleep**
  - Single positive numeric value giving the amount of time (in seconds) to `Sys.sleep` before drawing the next image in the animation.

- **override.par**
  - Logical value indicating whether to override the existing graphics device parameters prior to plotting, resetting `mfrow` and `mar`. See ‘Details’ for when you might want to disable this.

- **...**
  - Additional graphical parameters to be passed to `plot.im` (see ‘Details’).
rgmix

Details

Actual visualisation is deferred to \texttt{plot.im}, for which there are a variety of customisations available the user can access via \ldots

The \texttt{stim} object is plotted as an animation, one pixel image after another, separated by \texttt{sleep} seconds. If instead you intend the individual images to be plotted in an array of images, you should first set up your plot device layout, and ensure \texttt{override.par = FALSE} so that the function does not reset these device parameters itself. In such an instance, one might also want to set \texttt{sleep = 0}.

Value

Plots to the active graphics device.

Author(s)

T.M. Davies

Examples

\begin{verbatim}
# See help(stgmix) and help(stkey) for examples
\end{verbatim}

\begin{verbatim}

rgmix

\textit{Random bivariate Gaussian mixture density generation}

Description

Generates a pixel image of a bivariate normal mixture density observed on a bounded window using a specified number of contributing densities with randomly selected means and variance-covariance matrices.

Usage

\begin{verbatim}
rgmix(N, window, v = 4, S = NULL, extras = FALSE, ...)
\end{verbatim}

Arguments

\begin{verbatim}
N
window
v
S
extras
... 
\end{verbatim}

The number of Gaussian components to generate for the mixture.

An object of class \texttt{owin} giving the observational window on which the mixture density is defined.

The degrees of freedom for the inverse-Wishart distribution of the variance-covariance matrices (must be at least 4). The default value of 4 ensures the generated covariance matrices are centered on \texttt{S}.

A symmetric, positive-definite $2 \times 2$ scale matrix for the inverse-Wishart distribution of the variance-covariance matrices.

A logical value indicating whether, in addition to returning the pixel \texttt{image} of the final mixture density, to also return the randomly realised mean locations and corresponding variance-covariance matrices. See ‘Value’.

Additional arguments to be passed to \texttt{sgmix}. See ‘Details’.
Details

This function creates and returns a bivariate Gaussian mixture density on a bounded window based on \( N \) randomly generated mean locations and corresponding randomly generated variance-covariance matrices. First, the \( N \) mean locations are generated based on a uniform distribution over the spatial window. Each location is then associated with a covariance matrix generated from an inverse-Wishart distribution with \( v \) degrees of freedom and scale matrix \( S \).

Once the above steps are completed, the function calls \texttt{sgmix} with the chosen mean and covariance matrices, thereby creating the Gaussian mixture. Resolution and other aspects of this call can be controlled by using \ldots passing the contents internally to \texttt{sgmix}. By default, all generated Gaussian components have equal weight in contributing to the final mixture density. The user can alter this by passing \( p0 \) and \( p \) to the \ldots though should take care that the length of \( p \) is \( N \), and that \( p0 \) and \( p \) sum to 1, as outlined in the documentation for \texttt{sgmix}.

Value

If \texttt{extras = FALSE} (default), then a pixel \texttt{image} of the final mixture density. If \texttt{extras = TRUE}, a list is returned with members \texttt{f} (the pixel \texttt{image} of the final mixture density); \texttt{mn} (a \( 2 \times N \) matrix with each column giving the mean location of each of the \( N \) Gaussian bumps); and \texttt{vcv} (a \( 2 \times 2 \times N \) array with layers giving the covariance matrices associated with the means in the columns of \texttt{mn}).

Author(s)

A.K. Redmond and T.M. Davies

Examples

```r
set.seed(321)
dens1 <- rgmix(7, window=toywin)
plot(dens1)

set.seed(456)
dens2 <- rgmix(7, window=toywin)
plot(dens2)

# Explicitly return details of generated means and covariances
set.seed(321)
dens1.detailed <- rgmix(7, window=toywin, extras=TRUE)
dens1.detailed$f
dens1.detailed$mn
dens1.detailed$vcv

# Set underlying uniform proportion and compare with dens2 from above
set.seed(456)
dens2.wunif <- rgmix(7, window=toywin, p0=0.3)
par(mfrow=c(1,2))
plot(rpoint(500, dens2))
plot(rpoint(500, dens2.wunif))

# Explicitly setting scale matrix for inverse-Wishart generation of covariances
dens3 <- rgmix(3, window=toywin, S=matrix(c(0.025,-0.004,-0.004,0.02),2))
```
rpoispoly

plot(dens3)

rpoispoly

Generate a Poisson point pattern in a polygonal window

Description

Generates a single realisation of a spatial Poisson point process based on a pixel image and a polygonal window.

Usage

rpoispoly(z, w = NULL, correction = 1.1, maxpass = 50)

Arguments

z  A pixel image of class im defining the spatial intensity function of the points. The number of points generated, n, will be found as a randomly generated Poisson variate with mean parameter equal to the integral of z.

w  A polygonal window of class owin. See ‘Details’.

correction  An adjustment to the number of points generated at the initial pass of the internal loop in an effort to minimise the total number of passes required to reach n points. See ‘Details’ and ‘Warning’.

maxpass  The maximum number of passes allowed before the function exits. If this is reached before n points are found that fall within w, a warning is issued.

Details

This is a wrapper function for rimpoly that operates much like rpoispp, but with artificial corrections at the edges of boundary pixels. This allows the user to generate a realisation of a 2D Poisson point process using a supplied pixel image as the spatial intensity function, but return the result with a polygonal owin instead of a binary image mask.

Let n be a randomly generated integer from a Poisson distribution with mean given by the integral of the intensity function z. When the user specifies their own polygonal window in w, a while loop is called and repeated as many times as necessary (up to maxpass times) to find n points inside w (when w = NULL, then the union of the pixels of z is used, obtained via as.polygonal(Window(z))). The loop is necessary because the standard behaviour of rpoispp can (and often does) yield points that sit in corners of pixels which lie outside a corresponding irregular polygon w.

The correction argument is used to determine how many points are generated initially, which will be ceiling(correction*n); to minimise the number of required passes over the loop this is by default set to give a number slightly higher than the requested n.

An error is thrown if Window(z) and w do not overlap.

Value

An object of class ppp containing the Poisson-generated points, defined with the polygonal owin, w.
Warning

Note that this is an artificial correction that forces the Poisson-generated number of \( n \) points to be found inside any supplied polygon \( w \) (even if \( w \) only partially covers the domain of \( z \)). As such, this function only makes sense in terms of the theory of a Poisson point process if the polygon \( w \) corresponds exactly to the pixellised intensity. For practical intents and purposes, it therefore must be assumed in using this function that a supplied polygon \( w \) is/was the original basis for the discretisation into the pixel image for the purposes of producing the intensity \( z \), and hence that any adverse effects arising from imposing \( w \) as the window of the final result are negligible. See ‘Examples’.

Author(s)

T.M. Davies

References


See Also

rpoint, rimpoly, rpoispp

Examples

```
mn <- cbind(c(0.25,0.8),c(0.31,0.82),c(0.43,0.64),c(0.63,0.62),c(0.21,0.26))
v1 <- matrix(c(0.0023,-0.0009,-0.0009,0.002),2)
v2 <- matrix(c(0.0016,0.0015,0.0015,0.004),2)
v3 <- matrix(c(0.0007,0.0004,0.0004,0.0011),2)
v4 <- matrix(c(0.0023,-0.0041,-0.0041,0.0099),2)
v5 <- matrix(c(0.0013,0.0011,0.0011,0.0014),2)
vr <- array(NA,dim=c(2,2,5))
for(i in 1:5) vr[,i] <- get(paste("v",i,sep=""))
intens <- sgmix(mean=mn,vcv=vr,window=toywin,p0=0.1,int=500)

aa <- rpoispp(intens) # Default spatstat function
bb <- rpoispp(intens) # No polygon supplied; just uses pixel union
cc <- rpoispp(intens,w=toywin) # Original irregular polygon

par(mfrow=c(2,2))
plot(intens,log=TRUE)
plot(aa,main=paste("aa\nnn ",npoints(aa)))
plot(bb,main=paste("bb\nnn ",npoints(bb)))
plot(cc,main=paste("cc\nnn ",npoints(cc)))
```
rrmix

Spatial relative risk surface generation

Description
Generates an appropriately scaled spatial (bivariate) relative risk surface using a supplied control density and \( N \) isotropic Gaussian-style hotspots.

Usage

\texttt{rrmix(g, rhotspots, rsds, rweights, rbase = 1, log = TRUE)}

Arguments

- \texttt{g}: A pixel image representing the control density; this will be internally rescaled to integrate to 1 if it does not already do so.
- \texttt{rhotspots}: A \( 2 \times N \) matrix giving the centers of the \( N \) peaks and troughs in the relative risk density.
- \texttt{rsds}: A positive numeric vector of length \( N \) giving the isotropic standard deviations for each relative Gaussian peak or trough.
- \texttt{rweights}: A vector of length \( N \) giving relative weightings for each peak (positive weights) or trough (negative).
- \texttt{rbase}: The base level of the relative risk surface (default is 1). The peaks and troughs will be added or subtracted from this base level prior to normalisation.
- \texttt{log}: A logical value. If \texttt{TRUE} (default), the relative risk surface is returned logged.

Details
A useful tool for the comparison of two estimated density functions on the same spatial region \( W \subset \mathbb{R}^2 \) is the relative risk function, \( r \), (Bithell, 1990; 1991; Kelsall and Diggle, 1995), defined simply as a density-ratio:

\[
r(x) = f(x)/g(x); \ x \in W.
\]

Various methods have been developed to improve estimation of \( r \), most commonly with a motivation in geographical epidemiology, where the ‘numerator’ density \( f \) pertains to the observed disease cases and the ‘denominator’ density \( g \) reflects the distribution of the at-risk controls (Kelsall and Diggle, 1995; Hazelton and Davies, 2009; Davies and Hazelton, 2010). To test newly developed methodology, simulations based on known relative risk scenarios are usually necessary. This function allows the user to design such scenarios, as used in Hazelton and Davies (2009), Davies and Hazelton (2010), and Davies (2013) for example.

This function calculates a relative risk surface based on \( N \) Gaussian-style ‘bumps’ added and subtracted from a base level of \texttt{rbase}, with the peaks and troughs centered at the coordinates given by \texttt{rhotspots} with relative weights of \texttt{rweights} and isotropic standard deviations of \texttt{rsds}. The risk surface \( r \) is computed as

\[
r(x) \propto rbase + \sum_{i=1}^{N} rweights[i] \cdot \exp(-0.5 \cdot rsds[i] \cdot \|x - rhotspots[,i]\|^2)
\]
where \( \| \cdot \| \) denotes Euclidean norm. Because \( f \) and \( g \) are both densities, the risk surface as defined above must then be rescaled with respect to the supplied control density \( g \) (argument \( g \)) to ensure that
\[
\int_W r(x)g(x)dx = 1
\]
This is automatically performed inside the function. The case density that gives rise to the designed \( r \) is then easily recovered because \( f = r \ast g \). By default, the function returns the log-relative risk surface \( \log r = \log f - \log g \) alongside the case and control densities.

Value
An object of class \( \text{rrim} \). This is a \text{solist} of three pixel images: \( f \) as the case density, \( g \) the control density (a copy of the argument of the same name, integrating to 1), and \( r \) as the (log) relative risk surface.

Author(s)
A.K. Redmond and T.M. Davies

References


Examples
\begin{verbatim}
set.seed(1)
gg <- rgmix(3, window=toywin, S=matrix(c(0.08^2,0,0,0.1^2),nrow=2),p0=0.2)

rho <- rrmix(g=gg,
            rhotspots=cbind(c(0.8,0.3),c(0.4,0.4),c(0.6,0.5),c(0.3,0.5)),
            rsds=c(0.005,0.025,0.01,0.025),
            rweights=c(3,2,10,5)*10)

table <- rrtable(c(400,800),rho,toywin)

par(mfrow=c(2,2))
\end{verbatim}
**Description**

Generates a pair of random, independent point patterns corresponding to a case density and a control density, for relative risk analyses.

**Usage**

```
rrpoint(n, r, W = NULL, correction = 1.1, maxpass = 50)
rrstpoint(n, r, W = NULL, correction = 1.5, maxpass = 50)
```

**Arguments**

- `n` The number of points to be generated. This must be a numeric vector of length 2 giving the number of points to generate for the case and control densities respectively. Alternatively a single number can be supplied; then the same number of points is generated for both densities.
- `r` The relative risk surface object containing the definitions of the case and control probability densities: an object of class `rrim` or `rrs` for `rrpoint`, or an object of class `rrstim` or `rrst` for `rrstpoint`.
- `W` The polygonal `owin` defining the spatial window on which the density is defined. If `NULL`, this will be set to the `as.polygonal` version of the pixel images stored in `r`. See ‘Details’.
- `correction` An adjustment to the number of points generated at the initial pass of the internal loop in an effort to minimise the total number of passes required to reach `n` points.
- `maxpass` The maximum number of passes allowed before the function exits. If this is reached before `n` points are found with respect to the spatial or spatiotemporal domains of `r`, a warning is issued.

**Details**

These functions randomly generate a pair of independent spatial or spatiotemporal point patterns of `n` points based on the case and control density functions stored in `r`. At any given pass for each density, `n * correction` points are generated and rejection sampling is used to accept some of the points; this is repeated until the required number of points is found.

The argument `W` is optional, but is useful when the user wants the spatial window of the resulting point pattern to be a corresponding irregular polygon, as opposed to being based on the boundary of...
a binary image mask (which, when the pixel images in \( r \) are converted to a polygon directly, gives jagged edges based on the union of the pixels).

**Value**

A list with two components, cases and controls, each of which is an object of class \texttt{ppp} containing the \( n \) generated points. for spatiotemporal densities, the \texttt{marks} of the object will contain the correspondingly generated observation times.

**Author(s)**

T.M. Davies

**Examples**

# Using 'rrim' object:
```r
set.seed(1)
gg <- rgmix(3, window=toywin, S=matrix(c(0.08^2, 0, 0, 0.1^2), nrow=2), p0=0.2)
rho <- rmix(g=gg, rhotspots=cbind(c(0.8, 0.3), c(0.4, 0.4), c(0.6, 0.5), c(0.3, 0.5)), rsds=c(0.005, 0.025, 0.01, 0.025), rweights=c(3, 2, 10, 5)*10)
rho.sample <- rrpoint(n=c(400, 800), r=rho, W=toywin)
```

par(mfrow=c(2,2))
```r
plot(rho$g, main="control density")
plot(rho$f, main="case density")
plot(rho$r, main="log relative risk surface")
plot(rho.sample$controls, main="sample data")
points(rho.sample$cases, col=2)
legend("topright", col=2:1, legend=c("cases", "controls"), pch=1)
```

# Using 'rrs' object:
```r
require("sparr")
data(pbc)
pbccas <- split(pbc)$case
pbcon <- split(pbc)$control
h0 <- OS(pbc, nstarp=geometric)
f <- bivariate.density(pbccas, h0=h0, hp=2, adapt=TRUE, pilot.density=pbccas, edge="diggle", davies.baddeley=0.05, verbose=FALSE)
g <- bivariate.density(pbcon, h0=h0, hp=2, adapt=TRUE, pilot.density=pbccas, edge="diggle", davies.baddeley=0.05, verbose=FALSE)
pbcrr <- risk(f, tolerate=TRUE, verbose=FALSE)
```
```r
pbcrr.pt <- rrpoint(n=1000, r=pbcrr)
```

par(mfrow=c(1,3))
```r
plot(pbcrr)
plot(pbcrr.pt$cases)
```
rrstmix

Spatiotemporal relative risk surface generation

Description

Generates an appropriately scaled spatiotemporal (trivariate) relative risk surface using a supplied control density and \( N \) Gaussian-style hotspots.

Usage

\texttt{rrstmix(g, rhotspots, rsds, rweights, rbase = 1, log = TRUE,}
\texttt{ tlim = \text{NULL}, tres = \text{NULL})}
Arguments

\( g \)  
The control density as a \texttt{stim}, \texttt{stden}, or \texttt{im} object; this will be internally rescaled to integrate to 1 if it does not already do so. When a \texttt{stim} or \texttt{stden} object, the resolution and domain of the final result will be the same as this. When this argument is passed an object of class \texttt{im}, the function assumes a static (unchanging) control density over time (see Fernando and Hazelton, 2014), and the user must additionally specify \texttt{tlim} and \texttt{tres}.

\( \text{rhotspots} \)  
A \( 3 \times N \) matrix specifying the spatiotemporal coordinates of the \( N \) peaks and troughs in the relative risk density. The three entries down each column will be respectively interpreted as \( x \)-coord., \( y \)-coord., and time-coordinate of each Gaussian bump.

\( \text{rsds} \)  
A \( 3 \times N \) strictly positive numeric matrix specifying the standard deviations along each axis of each of the \( N \) bumps, the ordering of the components in each column is the same as \texttt{rhotspots}.

\( \text{rweights} \)  
A vector of length \( N \) giving relative weightings for each peak (positive weight) or trough (negative).

\( \text{rbase} \)  
The base level of the relative risk surface (default is 1). The peaks and troughs will be added or subtracted from this base level prior to normalisation.

\( \text{log} \)  
A logical value. If \texttt{TRUE} (default), the relative risk surface is returned logged.

\( \text{tlim} \)  
Only used if \( g \) is a pixel \texttt{image} object. A vector of length 2 giving the boundaries of the time interval on which the relative risk surface will be defined.

\( \text{tres} \)  
Only used if \( g \) is a pixel \texttt{image} object. The resolution along the temporal axis of the final result.

Details

This function is the spatiotemporal (trivariate) equivalent of \texttt{rrmix}. See ‘Details’ in the documentation for that function for more information.

Value

An object of class \texttt{rrstim}. This is a list with the following components:

\( f \)  
An object of class \texttt{stim} giving the case density.

\( g \)  
A copy of the object passed to the argument \( g \), possibly renormalised to integrate to 1 if this was necessary. If \( g \) was originally an \texttt{im}, this will be converted to an object of class \texttt{stim}.

\( r \)  
An object of class \texttt{stim} giving the (log) relative risk surface.

Author(s)

A.K. Redmond and T.M. Davies

References

Examples

# time-varying control density
gg1 <- stgmix(mean=matrix(c(2,1,3,0,-1,5),nrow=3),
              vcv=array(c(1,0,0,1,0,0,1,2,0,0,0,1,0,0,0,2),dim=c(3,3)),
              window=shp2, tlim=c(0,6))
rsk1 <- rrrstmix(g=gg1, rhotspots=matrix(c(-2,0,2,1,2,5,5),nrow=3),
              rsds=sqrt(cbind(rep(1.5,3), rep(0.25,3))), rweights=c(-0.5,5))
plot(rsk1$g, sleep=0.1, fix.range=TRUE)
plot(rsk1$f, sleep=0.1, fix.range=TRUE)
plot(rsk1$r, sleep=0.1, fix.range=TRUE)

# time-constant control density
set.seed(321)
gg2 <- rgmix(7, window=shp2)
rsk2 <- rrrstmix(g=gg2, rhotspots=matrix(c(-1,-1,2,2,5,0,5),nrow=3),
              rsds=sqrt(cbind(rep(0.75,3), c(0.05,0.01,0.5))),
              rweights=c(-0.4,7), tlim=c(0,6), tres=64)
plot(rsk2$g, sleep=0.1, fix.range=TRUE)
plot(rsk2$f, sleep=0.1, fix.range=TRUE)
plot(rsk2$r, sleep=0.1, fix.range=TRUE)

rstpoint

Generate random points in space-time

Description

Generates a random spatiotemporal point pattern containing \( n \) independent, identically distributed points with a specified distribution.

Usage

rstpoint(n, f, W = NULL, correction = 1.5, maxpass = 50)

Arguments

- **n**: The number of points to be generated.
- **f**: The probability density of the points, an object of class `stim` or `stden`.
- **W**: The polygonal `owin` defining the spatial window on which the density is defined. If `NULL`, this will be set to the `as.polygonal` version of the pixel images stored in `f`. See ‘Details’.
- **correction**: An adjustment to the number of points generated at the initial pass of the internal loop in an effort to minimise the total number of passes required to reach \( n \) points.
- **maxpass**: The maximum number of passes allowed before the function exits. If this is reached before \( n \) points are found with respect to the spatiotemporal domain of `f`, a warning is issued.
Details

This function randomly generates a spatiotemporal point pattern of exactly \( n \) points based on the density function \( f \). At any given pass, \( n \times \text{correction} \) points are generated and rejection sampling is used to accept some of the points; this is repeated until the required number of points is found.

The argument \( W \) is optional, but is useful when the user wants the spatial window of the resulting point pattern to be a corresponding irregular polygon, as opposed to being based on the boundary of a binary image mask (which, when the pixel images in \( f \) are converted to a polygon directly, gives jagged edges based on the union of the pixels).

Value

An object of class \texttt{ppp} containing the \( n \) generated points. The \texttt{marks} of the object contain the correspondingly generated observation times.

Author(s)

A.K. Redmond and T.M. Davies

Examples

```r
r1a <- sgmix(cbind(c(0.5,0.5)),vcv=0.01,window=toywin,p0=0.5,p=c(0.5),res=128)
r1b <- sgmix(cbind(c(0.5,0.5),c(0.4,0.6)),vcv=c(0.06,0.015),window=toywin, 
p0=0.1,p=c(0.5,0.4),res=128)
r1c <- sgmix(cbind(c(0.4,0.6)),vcv=c(0.1),window=toywin,p0=0.1,p=c(0.9),res=128)
sts1 <- stkey(start=r1a, 
    stop=r1c, 
    tlim=c(1,10), 
    tres=64, 
    window=toywin, 
    kf=solist(r1a,r1b), 
    kftimes=c(2,6), 
    fscale=0.1+0.9*dnorm(seq(-3,3,length=64),mean=0,sd=1))
plot(sts1,sleep=0.1)

Y <- rstpoint(500,sts1,W=toywin,correction=10,maxpass=500)
plot(Y)

require("rgl")
plot3d(Y$x,Y$y,marks(Y))
```

sgmix

\textit{Bivariate Gaussian mixture density generation}

Description

Generates a pixel image of a specified bivariate normal mixture density observed on a bounded window.
Usage

sgmix(mean, vcv, window, p0 = 0, p = NULL, resolution = 128, int = 1)

Arguments

mean
A $2 \times N$ matrix specifying the means of each of $N$ contributing normal densities.

vcv
Either a $2 \times 2 \times N$ array specifying the variance-covariance matrices of each contributing density, or a numeric vector of length $N$ giving the isotropic standard deviations of each contributing density. An error is thrown if the function encounters anything but a symmetric, positive-definite covariation specification for each component.

window
An object of class owin giving the observational window on which the mixture density is defined.

p0
The proportion of uniform density that contributes to the mixture (default is 0).

p
A numeric vector of the $N$ proportions for each contributing density (default is equal proportions for each density, after subtracting $p0$). Together, $p0$ and $p$ must sum to exactly 1.

resolution
The number of pixels along each side of the grid for the pixel image (default is 128).

int
A positive numeric value for post-hoc rescaling of the density (useful if the user wishes to return an intensity function). Defaults to 1 for no change in scaling.

Details

This function generates a pixel image of a 2D density function made of a mixture of $N$ bivariate normals; each component is restricted to conserve probability mass over a bounded subset of the plane. A warning will appear if less than 1% of the integral of each Gaussian bump is inside the observational window.

Value

An object of class im giving the mixture density.

Author(s)

A.K. Redmond

Examples

# Example using isotropic standard deviations
m1 <- c(0.4, 0.5)
m2 <- c(0.2, 0.7)
s1 <- 0.1
s2 <- 0.025
dens1 <- sgmix(mean=cbind(m1, m2), vcv=c(s1, s2), window=toywin, p0=0.3, p=c(0.5, 0.2))

plot(dens1, log=TRUE)
pts1 <- rpoint(200, dens1) # generate random points via spatstat::rpoint
stgmix

Example using full covariance matrices

```r
# Example using full covariance matrices

mn <- cbind(c(0.25, 0.8), c(0.31, 0.82), c(0.43, 0.64), c(0.63, 0.62), c(0.21, 0.26))

v1 <- matrix(c(0.0023, -0.0009, -0.0009, 0.002), 2)
v2 <- matrix(c(0.0016, 0.0015, 0.0015, 0.004), 2)
v3 <- matrix(c(0.0007, 0.0004, 0.0004, 0.0011), 2)
v4 <- matrix(c(0.0043, -0.0041, -0.0041, 0.0099), 2)
v5 <- matrix(c(0.0013, 0.0011, 0.0011, 0.0014), 2)

vr <- array(NA, dim=c(2,2,5))
for(i in 1:5) vr[,,i] <- get(paste("v",i,sep=""))

dens2 <- stgmix(mean=mn, vcv=vr, window=toywin, p0=0.1)

plot(dens2, log=TRUE)
pts2 <- rpoint(200, dens2)
points(pts2)
```

---

### Description

Generates a pixel image array of a specified trivariate normal mixture density observed on a bounded window in space and time.

### Usage

```r
stgmix(mean, vcv, window, tlim, p0 = 0, p = NULL, sres = 128, tres = sres, int = 1)
```

### Arguments

- **mean**
  - A $3 \times N$ matrix specifying the means of each of $N$ contributing normal densities; each component in the order of (x-coord, y-coord, time-coord).

- **vcv**
  - A $3 \times 3 \times N$ array specifying the variance-covariance matrices of each contributing density.

- **window**
  - An object of class `owin` giving the spatial observational window on which the mixture density is defined.

- **tlim**
  - A vector of length 2 giving the boundaries of the time interval on which the mixture density is defined.

- **p0**
  - The proportion of uniform density that contributes to the final mixture (default is 0).

- **p**
  - A numeric vector of the $N$ proportions for each contributing density (default is equal proportions for each density, after subtracting $p0$). Together, $p0$ and $p$ must sum to exactly 1.

- **sres**
  - The spatial resolution (number of pixels) along each side of the spatial grid (default is 128).
 tres The temporal resolution (default is to equate with sres).
 int A positive numeric value for post-hoc rescaling of the density (useful if the user wishes to return a spatiotemporal intensity function). Defaults to 1 for no change in scaling.

Details

This function creates a 3D array of a density function made up of a mixture of $N$ trivariate normals with the interpretation of a continuous probability density function in space-time. As such, each component is restricted to conserve mass over a 3D region specified by a fixed polygonal window in space, stretched over defined temporal limits (tlim). A warning will appear if less than 1% of the integral of each Gaussian bump is inside this observational spatiotemporal polyhedron.

Value

An object of class stim giving the trivariate density. This is a list with six components:

 a The sres x sres x tres array of the specified density.
 v A pixel image version of a, provided as a solist of length tres, with each member being the spatial image slice of the 3D density at each of the time-coordinate values.
 xcol Grid coordinates in the spatial x-axis (corresponds to each spatial image in v).
 yrow Grid coordinates in the spatial y-axis (corresponds to each spatial image in v).
 tlay Grid coordinates in the temporal axis (corresponds to the order of the spatial images in v).
 w A copy of window, the spatial owin upon which the density is defined.

Author(s)

A.K. Redmond and T.M. Davies

Examples

```r
require("abind")
m1 <- c(0.3,0.3,2)
m2 <- c(0.5,0.8,8)
m3 <- c(0.7,0.6,7)
v1 <- diag(c(0.01^2,0.01^2,1))
v2 <- diag(c(0.005,0.005,0.5))
v3 <- diag(c(0.005,0.005,0.5))
stg1 <- stgmix(mean=cbind(m1,m2,m3),
    vcv=abind(v1,v2,v3,along=3),
    window=toywin,tlim=c(1,10),
    p0=0.1,tres=64)
plot(stg1,log=TRUE)

mm <- matrix(c(0,0,0,-2,1,4,1,-2,8),nrow=3)
```


vr <- array(c(1,0,0,1,0,0,0,1,1,0,0,0.5,0,1,0,0.5,0,3,1,0,0,0,2,0,0,0,1),
            dim=c(3,3,3))
stg2 <- stgmix(mean=mn,vcv=vr,window=shp1,
               tlim=c(0,10),tres=50)
plot(stg2,fix.range=TRUE,sleep=0.1)

---

**stintegral**

Evaluate integral of a spatiotemporal object

**Description**

Integrates an object of class *stim* or *stden*.

**Usage**

```r
stintegral(x)
```

**Arguments**

- `x` The object of class *stim* or *stden* to be integrated.

**Details**

The integral is evaluated arithmetically as the sum of the product of the value of each voxel and the voxel area, for those voxels inside the relevant space-time window (i.e. ignoring NAs).

**Value**

A single numeric value giving the integral sought.

**Author(s)**

T.M. Davies

**Examples**

```r
# 'stim' objects
require("abind")
m1 <- c(0.3,0.3,2)
m2 <- c(0.5,0.8,8)
m3 <- c(0.7,0.6,7)
v1 <- diag(c(0.01^2,0.01^2,1))
v2 <- diag(c(0.005,0.005,0.5))
v3 <- diag(c(0.005,0.005,0.5))
stg1 <- stgmix(mean=cbind(m1,m2,m3),vcv=abind(v1,v2,v3,along=3),
               window=toywin,tlim=c(0,10),p8=0.1,tres=64)
stg2 <- stgmix(mean=cbind(m1,m2,m3),vcv=abind(v1,v2,v3,along=3),
               window=toywin,tlim=c(0,10),p8=0.1,tres=64)```
Spatiotemporal density generation via keyframe interpolation

Description

Uses the supplied spatial pixel images and scalings to linearly interpolate the behaviour of the function over time, creating a trivariate density function in space-time.

Usage

```r
stkey(start, stop, tlim, kf = NULL, tres = 64,
      kftimes = NULL, fscales = NULL, window = NULL)
```

Arguments

- **start**: The spatial pixel image corresponding to the spatial density at start of the time interval. May be unnormalised, the function internally rescales all supplied spatial images to integrate to 1.
- **stop**: The pixel image for the end of the time interval. Must be compatible with `start`, in that it is defined over the same spatial domain and is of identical resolution.
- **tlim**: A numeric vector of length 2 representing the temporal window i.e. the time interval over which the interpolation takes place.
- **kf**: A `solist` of the pixel images of the keyframes between `start` and `stop`. If supplied, each image must be compatible with `start` and `stop`. If unsupplied, the resulting interpolation is performed only on `start` and `stop`.
- **tres**: The resolution of the resulting array in the time dimension (default is 64).
- **kftimes**: A vector of times that position the interim keyframes in `kf` between `tlim[1]` and `tlim[2]`. Ignored if `kf = NULL`. If unsupplied (NULL), but `kf` is present, the function simply positions the images of `kf` at evenly spaced time points in the `tlim` interval.
- **fscales**: A numeric vector of unnormalised, relative point-intensity scales. This may be provided either as of `length(kf) + 2`, so the point intensities assinged to each frame in the order `c(start, entries of kf, stop)`, or of length `tres`. If unsupplied, each spatial frame is simply given equal weight. See ‘Details’.
window An object of class owin giving the polygonal spatial observational window on which the density is defined. If NULL, the polygon is simply obtained from the union of pixel values in the supplied images.

Details

This function interpolates in a pixel-wise fashion between the im objects supplied as start and stop (and kf if supplied), placing them as keyframes at the times in tlim (for start and stop) and kftimes (for the members of kf). The final result is rescaled such that its total integrated volume over the defined spatiotemporal domain is 1, yeilding a trivariate density function.

If fscale is a vector of length tres, each element will correspond to the relative overall scaling of one of the resulting interpolated pixel images. If it is of length length(kf) + 2, the scales will correspond to start, each keyframe in kf and stop in that order. The values in this argument are only interpretable in a relative sense: for example, with a single keyframe supplied to kf (in addition to the required start and stop), then fscales = c(0.5, 1, 0.5) has exactly the same effect on the final result as fscales = c(1, 1, 1), and is interpreted as yielding a point density that reaches twice the concentration at the time of the supplied keyframe relative to the start and stop margins. Supplying fscale as a vector of length tres thus allows finer control over the relative point density over time, such as for the incorporation of harmonic seasonal variation.

Value

Like stgmix, an object of class stim giving the trivariate density. This is a list with six components:

a The xr x yr x tres array of the specified density (where xr and yr are used here to denote the spatial resolution along the x- and y-axes; this is governed in stkey by the images initially supplied to start and stop).

v A pixel image version of a, provided as a solist of length tres, with each member being the spatial image slice of the 3D density at each of the time-coordinate values.

xcol Grid coordinates in the spatial x-axis (corresponds to each spatial image in v).

yrow Grid coordinates in the spatial y-axis (corresponds to each spatial image in v).

tlay Grid coordinates in the temporal axis (corresponds to the order of the spatial images in v).

W A copy of window, the spatial owin upon which the density is defined.

Author(s)

A.K. Redmond and T.M. Davies

Examples

```r
mn <- matrix(c(0,0,1,2,0.5,-1),nrow=2)
vr <- array(c(0.2,0,0,2,1,0,0,1,1,0.3,0.5),dim=c(2,2,3))
im1 <- sgmix(mn,c(1,2,1),shp1=p=c(0.4,0.3,0.3))
im2 <- sgmix(matrix(c(-3,0,0,-2,-1,2),nrow=2),c(3,1,1),shp1,p=c(0.4,0.3,0.3))
im3 <- sgmix(mn,vr,shp1,p0=0.1)
```
### toywin

**Toy Windows**

**Description**

Synthetic spatial windows for use in testing, simulations and demonstrations.

**Usage**

```r
data(bx)
data(heart)
data(shp1)
data(shp2)
data(star)
data(toywin)
```

**Format**

Each of these is a single closed polygon of class `owin`.

- `bx` is a box on $[-5,5]^2$.
- `heart` is a heart, professing love for all things `spatstat`.
- `shp1` is shape of mystery.
- `shp2` is a slightly more symmetric shape of mystery.
- `star` is a star that shines brightly in even non-spatial contexts.
- `toywin` is the eponymous toy window used in publications e.g. Davies & Lawson (2018).

**Details**

These are lazy-loaded so may be called directly by name upon loading of `spagmix`.
Author(s)

A.K. Redmond and T.M. Davies

References


Examples

```r
par(mfrow=c(2,3))
plot(bx);axis(1);axis(2)
plot(heart);axis(1);axis(2)
plot(shp1);axis(1);axis(2)
plot(shp2);axis(1);axis(2)
plot(star);axis(1);axis(2)
plot(toywin);axis(1);axis(2)
```

---

**unify.owin**

**Spatial window unit rescaler**

Description

Rescales any owin to fall inside the unit square.

Usage

```r
unify.owin(W)
```

Arguments

- `W` An object of class owin giving the spatial window to be transformed.

Details

This function is a simple wrapper for `affine` deployed to rescale a supplied owin to fall inside the unit square.

Value

The rescaled owin.

Examples

```r
W <- Window(chorley)
U <- unify.owin(W)

par(mfrow=c(1,2))
plot(W,axes=TRUE)
plot(U,axes=TRUE)
```
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