

# Package ‘stabledist’

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**Title** Stable Distribution Functions

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**Depends** R (>= 2.12.0), stats, utils

**Suggests** Matrix, fBasics, FMStable, RUnit, Rmpfr, sfsmisc

**Description** Density, Probability and Quantile functions, and random number generation for (skew) stable distributions, using the parametrizations of Nolan.

**LazyData** yes

**License** GPL (>= 2)

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StableDistribution      *Stable Distribution Function*

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### Description

A collection and description of functions to compute density, distribution and quantile function and to generate random variates of the stable distribution.

The four functions are:

[dpqr]stable    the (skewed) stable distribution.

### Usage

```
dstable(x, alpha, beta, gamma = 1, delta = 0, pm = 0,
        log = FALSE,
        tol = 64*.Machine$double.eps, zeta.tol = NULL,
        subdivisions = 1000)
pstable(q, alpha, beta, gamma = 1, delta = 0, pm = 0,
        lower.tail = TRUE, log.p = FALSE,
        tol = 64*.Machine$double.eps, subdivisions = 1000)
qstable(p, alpha, beta, gamma = 1, delta = 0, pm = 0,
        lower.tail = TRUE, log.p = FALSE,
        tol = .Machine$double.eps^0.25, maxiter = 1000,
        integ.tol = 1e-7, subdivisions = 200)
rstable(n, alpha, beta, gamma = 1, delta = 0, pm = 0)
```

### Arguments

alpha, beta, gamma, delta	value of the index parameter alpha with $\alpha \in (0, 2]$ ; skewness parameter beta, in the range $[-1, 1]$ ; scale parameter gamma; and location (or 'shift') parameter delta.
n	sample size (integer).
p	numeric vector of probabilities.
pm	parameterization, an integer in $\{0, 1, 2\}$ ; by default $pm=0$ , the 'S0' parameterization.
x, q	numeric vector of quantiles.
log, log.p	logical; if TRUE, probabilities p are given as $\log(p)$ .
lower.tail	logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ .
integ.tol	positive number, the tolerance used for numerical integration, see <a href="#">integrate</a> .
tol	numerical tolerance, <b>dstable(), pstable():</b> used for numerical integration, see <code>integ.tol</code> above. Note that earlier versions had tighter tolerances – which seem too tight as default values.

	<b>qstable()</b> : used for rootfinding, see <a href="#">uniroot</a> .
zeta.tol	(dstable) numerical tolerance for checking if x is close to $\zeta(\alpha, \beta)$ . The default, NULL depends itself on $(\alpha, \beta)$ . As it is <i>experimental</i> and not guaranteed to remain in the future, its use is not recommended in production code. Rather e-mail the package maintainer about it.
subdivisions	maximal number of intervals for integration, see <a href="#">integrate</a> .
maxiter	maximal number of iterations in <a href="#">uniroot</a> .

## Details

### Skew Stable Distribution:

The function uses the approach of J.P. Nolan for general stable distributions. Nolan (1997) derived expressions in form of integrals based on the characteristic function for standardized stable random variables. For `dstable` and `pstable`, these integrals are numerically evaluated using R's `integrate()` function.

“S0” parameterization [pm=0]: based on the (M) representation of Zolotarev for an alpha stable distribution with skewness beta. Unlike the Zolotarev (M) parameterization, gamma and delta are straightforward scale and shift parameters. This representation is continuous in all 4 parameters, and gives an intuitive meaning to gamma and delta that is lacking in other parameterizations. Switching the sign of beta *mirrors* the distribution at the vertical axis  $x = \delta$ , i.e.,

$$f(x, \alpha, -\beta, \gamma, \delta, 0) = f(2\delta - x, \alpha, +\beta, \gamma, \delta, 0),$$

see the graphical example below.

“S” or “S1” parameterization [pm=1]: the parameterization used by Samorodnitsky and Taqqu in the book *Stable Non-Gaussian Random Processes*. It is a slight modification of Zolotarev’s (A) parameterization.

“S\*” or “S2” parameterization [pm=2]: a modification of the S0 parameterization which is defined so that (i) the scale gamma agrees with the Gaussian scale (standard dev.) when alpha=2 and the Cauchy scale when alpha=1, (ii) the mode is exactly at delta. For this parametrization, `stableMode(alpha, beta)` is needed.

“S3” parameterization [pm=3]: an internal parameterization, currently not available for these functions. The scale is the same as the “S2” parameterization, the shift is  $-\beta * g(\alpha)$ , where  $g(\alpha)$  is defined in Nolan(1999).

## Value

All values for the `*stable` functions are numeric vectors: `d*` returns the density, `p*` returns the distribution function, `q*` returns the quantile function, and `r*` generates random deviates.

## Tail Behavior

The asymptotic behavior for large  $x$ , aka “tail behavior” for the cumulative  $F(x) = P(X \leq x)$  is (for  $x \rightarrow \infty$ )

$$1 - F(x) \sim (1 + \beta)C_\alpha x^{-\alpha},$$

where  $C_\alpha = \Gamma(\alpha)/\pi \sin(\alpha\pi/2)$ ; hence also

$$F(-x) \sim (1 - \beta)C_\alpha x^{-\alpha}.$$

Differentiating  $F()$  above gives

$$f(x) \sim \alpha(1 + \beta)C_\alpha x^{-(1+\alpha)}.$$

### Note

In the case  $\beta = 1$ , the distributions are “maximally skewed to the right” or simply “*extremal stable*” (Zolotarev). In that case, the package **FMStable** provides `dpq*` functions which are faster and more accurate than ours (if accuracy higher than about 6 digits is needed), see, [pEstable](#).

When alpha is close to 1 or close to 0 (“close”, e.g., meaning distance  $d < 0.01$ ), the computations typically are numerically considerably more challenging, and the results may not be accurate.

As we plan to improve on this, *and* as it is unknown when exactly the numerical difficulties arise, we currently only do warn here (in the documentation), but *not* by giving explicit `warning()`s.

### Author(s)

Diethelm Wuertz for the original Rmetrics R-port. Many numerical improvements by Martin Maechler.

### References

Chambers J.M., Mallows, C.L. and Stuck, B.W. (1976) *A Method for Simulating Stable Random Variables*, J. Amer. Statist. Assoc. **71**, 340–344.

John P. Nolan (2012) *Stable Distributions - Models for Heavy Tailed Data* Birkhauser, Boston; in progress, chapter 1 online at <http://academic2.american.edu/~jpnolan/stable/chap1.pdf>

Nolan J.P. (1997) Numerical calculation of stable densities and distribution functions. *Stochastic Models* **13**(4), 759–774.

Also available as ‘density.ps’ from Nolan’s web page.

Samoridnitsky G., Taqqu M.S. (1994); *Stable Non-Gaussian Random Processes, Stochastic Models with Infinite Variance*, Chapman and Hall, New York, 632 pages.

Weron, A., Weron R. (1999); *Computer Simulation of Levy alpha-Stable Variables and Processes*, Preprint Technical University of Wroclaw, 13 pages.

### See Also

the `stableSlider()` function from package **fBasics** for displaying densities and probabilities of these distributions, for educational purposes.

### Examples

```
## stable -

## Plot stable random number series
set.seed(1953)
r <- rstable(n = 1000, alpha = 1.9, beta = 0.3)
```

```

plot(r, type = "l", main = "stable: alpha=1.9 beta=0.3",
     col = "steelblue")
grid()

## Plot empirical density and compare with true density:
hist(r, n = 25, probability = TRUE, border = "white",
     col = "steelblue")
x <- seq(-5, 5, 0.25)
lines(x, dstable(x, alpha = 1.9, beta = 0.3, tol= 1e-3), lwd = 2)

## Plot df and compare with true df:
plot(ecdf(r), do.points=TRUE, col = "steelblue",
     main = "Probabilities:  ecdf(rstable(1000,..)) and true  cdf F()")
rug(r)
lines(x, pstable(q = x, alpha = 1.9, beta = 0.3),
     col="#0000FF88", lwd= 2.5)

## Switching sign(beta) <==> Mirror the distribution around x == delta:
curve(dstable(x, alpha=1.2, beta = .8, gamma = 3, delta = 2), -10, 10)
curve(dstable(x, alpha=1.2, beta = -.8, gamma = 3, delta = 2),
     add=TRUE, col=2)
## or the same
curve(dstable(2*2-x, alpha=1.2, beta = +.8, gamma = 3, delta = 2),
     add=TRUE, col=adjustcolor("gray",0.2), lwd=5)
abline(v = 2, col = "gray", lty=2, lwd=2)
axis(1, at = 2, label = expression(delta == 2))

## Compute quantiles:
x. <- -4:4
px <- pstable(x., alpha = 1.9, beta = 0.3)
(qs <- qstable(px, alpha = 1.9, beta = 0.3))
stopifnot(all.equal(as.vector(qs), x., tol = 1e-5))

```

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StableMode

*Mode of the Stable Distribution Function*


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## Description

Computes the mode of the stable distribution, i.e., the maximum of its density function in the "0" parametrization, i.e., the maximum  $x_0$  of `dstable(x, alpha, beta, gamma = 1, delta = 0, pm = 0)`.

Finds the maximum of `dstable` numerically, using `optimize`.

## Usage

```

stableMode(alpha, beta,
           beta.max = 1 - 1e-11,
           tol = .Machine$double.eps^0.25)

```

**Arguments**

alpha, beta	numeric parameters: value of the index parameter alpha in the range $(0, 2]$ , and the skewness parameter beta, in the range $[-1, 1]$ .
beta.max	for numerical purposes, values of beta too close to 1, are set to beta.max. Do not modify unless you know what you're doing.
tol	numerical tolerance for <code>optimize()</code> .

**Value**

returns a numeric value, the location of the stable mode.

**Author(s)**

Diethelm Wuertz for the Rmetrics R-port; minor cleanup by Martin Maechler.

**See Also**

For definition and the “dpqr”-functions, [StableDistribution](#), also for the references.

**Examples**

```
## beta = 0 <==> symmetric <==> mode = 0
all.equal(stableMode(alpha=1, beta=0), 0)
al.s <- c(1e-100, seq(0,2, by = 1/32)[-1])
stopifnot(vapply(al.s, function(alp)
  stableMode(alpha=alp, beta=0), 1.) == 0)

## more interesting: asymmetric (beta != 0):
stableMode(alpha=1.2, beta=0.1)

if(stabledist:::doExtras()) { # takes 2.5 seconds
  sm0.5 <- vapply(al.s, function(AA)
    stableMode(alpha=AA, beta= 0.5), 1.)
  plot(al.s, sm0.5, type = "o", col=2, xlab = quote(alpha), ylab="mode",
    main = quote("Mode of stable"*{ }(alpha, beta == 0.5, pm==0))
  }
}
```

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