

# Package ‘biwavelet’

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**Type** Package

**Title** Conduct univariate and bivariate wavelet analyses

**Version** 0.17.3

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**Author** Tarik C. Gouhier, Aslak Grinsted

**Maintainer** Tarik C. Gouhier <tarik.gouhier@gmail.com>

**Imports** fields

## Description

This is a port of the WTC MATLAB package written by Aslak Grinsted and the wavelet program written by Christopher Torrence and Gibert P. Compo. This package can be used to perform univariate and bivariate (cross-wavelet, wavelet coherence, wavelet clustering) analyses.

**License** GPL (>= 2)

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biwavelet-package	<i>Conduct univariate and bivariate wavelet analyses</i>
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**Description**

This is a port of the WTC MATLAB package written by Aslak Grinsted and the wavelet program written by Christopher Torrence and Gibert P. Compo. This package can be used to perform univariate and bivariate (cross-wavelet, wavelet coherence, wavelet clustering) wavelet analyses.

**Details**

Package:	biwavelet
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Version:	0.17.3
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License:	GPL (>=2)
LazyLoad:	yes

As of biwavelet version 0.14, the bias-corrected wavelet and cross-wavelet spectra are automatically computed and plotted by default using the methods described by Liu et al. (2007) and Veeda et al. (2012). This correction is needed because the traditional approach for computing the power spectrum (e.g., Torrence and Compo 1998) leads to an artificial and systematic reduction in power at lower periods.

**Author(s)**

Tarik C. Gouhier

Maintainer: Tarik C. Gouhier <tarik.gouhier@gmail.com>

Code based on WTC MATLAB package written by Aslak Grinsted and the wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

**References**

- Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.
- Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.
- Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.
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- Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.
- Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.
- Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.
- Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

**Examples**

```
# As of biwavelet version 0.14, the bias-corrected wavelet and cross-wavelet spectra
# are automatically computed and plotted by default using the methods
# described by Liu et al. (2007) and Veleda et al. (2012). This correction
# is needed because the traditional approach for computing the power spectrum
# (e.g., Torrence and Compo 1998) leads to an artificial and systematic reduction
# in power at low periods.

# EXAMPLE OF BIAS CORRECTION:
require(biwavelet)
# Generate a synthetic time series 's' with the same power at three distinct periods
t1=sin(seq(from=0, to=2*5*pi, length=1000))
t2=sin(seq(from=0, to=2*15*pi, length=1000))
t3=sin(seq(from=0, to=2*40*pi, length=1000))
s=t1+t2+t3

# Compare non-corrected vs. corrected wavelet spectrum
wt1=wt(cbind(1:1000, s))
par(mfrow=c(1,2))
plot(wt1, type="power.corr.norm", main="Bias-corrected")
```

```

plot(wt1, type="power.norm", main="Not-corrected")

# Compare non-corrected vs. corrected cross-wavelet spectrum
x1=xwt(cbind(1:1000, s), cbind(1:1000, s))
par(mfrow=c(1,2))
plot(x1, type="power.corr.norm", main="Bias-corrected")
plot(x1, type="power.norm", main="Not-corrected")

# ADDITIONAL EXAMPLES
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
# Continuous wavelet transform
wt.t1=wt(t1)
# Plot power
# Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)

# Cross-wavelet
xwt.t1t2=xwt(t1, t2)
# Plot cross wavelet power and phase difference (arrows)
plot(xwt.t1t2, plot.cb=TRUE)

# Wavelet coherence; nrand should be large (>= 1000)
wtc.t1t2=wtc(t1, t2, nrand=10)
# Plot wavelet coherence and phase difference (arrows)
# Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wtc.t1t2, plot.cb=TRUE)

# Perform wavelet clustering of three time series
t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
t3=cbind(1:100, rnorm(100))
# Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
wt.t3=wt(t3)
# Store all wavelet spectra into array
w.arr=array(NA, dim=c(3, NROW(wt.t1$wave), NCOL(wt.t1$wave)))
w.arr[1, , ]=wt.t1$wave
w.arr[2, , ]=wt.t2$wave
w.arr[3, , ]=wt.t3$wave

# Compute dissimilarity and distance matrices
w.arr.dis=wclust(w.arr)
plot(hclust(w.arr.dis$dist.mat, method="ward"), sub="", main="",
      ylab="Dissimilarity", hang=-1)

```

**Description**

Generate the power spectrum of a random time series with a specific AR(1) coefficient

**Usage**

```
ar1.spectrum (ar1, periods)
```

**Arguments**

ar1                    first order coefficient desired.  
periods                periods of the time series at which the spectrum should be computed.

**Value**

Returns the power spectrum.

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)  
Code based on WTC MATLAB package written by Aslak Grinsted.

**References**

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

**Examples**

```
p=ar1.spectrum(0.5, 1:25)
```

---

check.data	<i>Check the format of time series</i>
------------	--

---

**Description**

Check the format of time series

**Usage**

```
check.data (y, x1 = NULL, x2 = NULL)
```

**Arguments**

y	time series y in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
x1	time series x1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
x2	time series x2 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.

**Value**

Returns a named list containing:

t	time steps
dt	size of a time step
n.obs	number of observations

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

**References**

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

**Examples**

```
# Not run:
# t1=cbind(1:100, rnorm(100))
# check.data(y=t1)
```

---

convolve2D

*Fast column-wise convolution of a matrix*


---

**Description**

Use the Fast Fourier Transform to perform convolutions between a sequence and each column of a matrix.

**Usage**

```
convolve2D (x, y, conj = TRUE, type = c("circular", "open"))
```

**Arguments**

x	M x n matrix.
y	numeric sequence of length N.
conj	logical; if TRUE, take the complex conjugate before back-transforming. Default is TRUE and used for usual convolution.
type	character; one of <code>circular</code> , <code>open</code> (beginning of word is ok). For <code>circular</code> , the two sequences are treated as circular, i.e., periodic. For <code>open</code> and <code>filter</code> , the sequences are padded with zeros (from left and right) first; <code>filter</code> returns the middle sub-vector of <code>open</code> , namely, the result of running a weighted mean of <code>x</code> with weights <code>y</code> .

**Details**

This is a corrupted version of `convolve` made by replacing `fft` with `mvfft` in a few places. It would be nice to submit this to the R Developers for inclusion.

**Note**

This function was copied from `waveslim` to limit package dependencies.

**Author(s)**

Brandon Whitcher

---

enviro.data

*Multivariate ENSO (MEI), NPGO, and PDO indices*

---

**Description**

Monthly indices of ENSO, NPGO, and PDO from 1950 to 2009

**Usage**

`data (enviro.data)`

**Format**

A data frame with 720 observations on the following 6 variables.

`month` a numeric vector containing the month  
`year` a numeric vector containing the year  
`date` a numeric vector containing the date  
`mei` a numeric vector containing the MEI index  
`npgo` a numeric vector containing the NPGO index  
`pdo` a numeric vector containing the PDO index

**Source**

MEI: <http://www.esrl.noaa.gov/psd/enso/mei>

NPGO: <http://www.o3d.org/npgo>

PDO: <http://jisao.washington.edu/pdo>

**References**

Di Lorenzo, E., N. Schneider, K. M. Cobb, P. J. S. Franks, K. Chhak, A. J. Miller, J. C. McWilliams, S. J. Bograd, H. Arango, E. Curchitser, T. M. Powell, and P. Riviere. 2008. North Pacific Gyre Oscillation links ocean climate and ecosystem change. *Geophys. Res. Lett.* 35:L08607.

Mantua, N. J., and S. R. Hare. 2002. The Pacific decadal oscillation. *Journal of Oceanography* 58:35-44.

Zhang, Y., J. M. Wallace, and D. S. Battisti. 1997. ENSO-like interdecadal variability: 1900-93. *Journal of Climate* 10:1004-1020.

**Examples**

```
data(enviro.data)
head(enviro.data)
```

---

meshgrid

*Rectangular grid in two dimensions*

---

**Description**

Replicates the grid vectors xv and yv to generate a full grid

**Usage**

```
meshgrid (xv, yv)
```

**Arguments**

xv                    vector of numeric values

yv                    vector of numeric values

**Value**

Returns a list containing the full grid components xv and yv:

x                    replicated values of xv

y                    replicated values of yv

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)



**Examples**

```
xv=runif(10)
yv=runif(5)
g=meshgrid(xv, yv)
```

---

phase.plot

*Plot phases*

---

**Description**

Plot phases with arrows

**Usage**

```
phase.plot(x, y, phases, arrow.size = 0.05, arrow.size.head = 0.05,
           arrow.lwd = 2, arrow.col="black")
```

**Arguments**

x	x-coordinates
y	y-coordinates
phases	phases
arrow.size	arrow size. Default is 0.05
arrow.size.head	size of the arrow head. Default is 0.05
arrow.lwd	arrow line width. Default is 2.
arrow.col	arrow line color. Default is black.

**Note**

Arrows pointing to the right mean that x and y are in phase.  
Arrows pointing to the left mean that x and y are in anti-phase.  
Arrows pointing up mean that y leads x by  $\pi/2$ .  
Arrows pointing down mean that x leads y by  $\pi/2$ .

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)  
Code based on WTC MATLAB package written by Aslak Grinsted.

**Examples**

```
## Not run: phase.plot(x, y, phases)
```

---

plot.biwavelet      *Plot biwavelet objects*

---

### Description

Plot biwavelet objects such as the cwt, cross-wavelet and wavelet coherence

### Usage

```
## S3 method for class 'biwavelet'
plot(x, ncol = 64, xlab = "Time", ylab = "Period",
     tol = 0.95, plot.cb = FALSE, plot.phase = FALSE,
     type=c("power.corr.norm", "power.corr", "power.norm", "power",
           "wavelet", "phase"),
     plot.coi = TRUE, lwd.coi=4, col.coi="white", lty.coi=2,
     plot.sig = TRUE, lwd.sig=4, col.sig="black", lty.sig=1,
     bw = FALSE, legend.loc = NULL, legend.horiz = FALSE,
     arrow.size = 0.08, arrow.size.head = 0.05, arrow.lwd = 2,
     arrow.cutoff = 0.9, arrow.col="black",
     arrow.nlocs.x = round(length(x$period)/2),
     arrow.nlocs.y = round(NCOL(x$wave)/2),
     xlim = NULL, ylim = NULL, xaxt = "s", yaxt = "s",
     form='%Y', ...)
```

### Arguments

x	biwavelet object generated by wt, xwt, or wtc.
ncol	number of colors to use. Default is 64.
xlab	xlabel of the figure. Default is "Time"
ylab	ylabel of the figure. Default is "Period"
tol	tolerance level for significance contours. Significance contours will be drawn around all regions of the spectrum where spectrum/percentile $\geq$ tol. Default is 0.95. If strict $i^{\text{th}}$ percentile regions are desired, then set tol=1.
plot.cb	plot color bar if TRUE. Default is FALSE.
plot.phase	Plot phases with black arrows. Default is FALSE.
type	type of plot to create. Can be power to plot the power, power.corr to plot the bias-corrected power, power.norm to plot the power normalized by the variance, power.corr.norm to plot the bias-corrected power normalized by the variance, wavelet to plot the wavelet coefficients, or phase to plot the phase. Default is power.corr.norm.
plot.coi	plot cone of influence (COI) if TRUE. Default is TRUE. Areas that fall outside of the COI can be affected by edge effects.
lwd.coi	Line width of COI. Default is 4.
col.coi	Color of cone of COI. Default is white.

lty.coi	Line type of COI
plot.sig	plot contours for significance if TRUE. Default is TRUE.
lwd.sig	Line width of significance contours. Default is 4.
col.sig	Color of significance contours. Default is black.
lty.sig	Line type of significance contours. Default is 1.
bw	plot in black and white if TRUE. Default is FALSE.
legend.loc	legend location coordinates as defined by image.plot. Default is NULL.
legend.horiz	plot a horizontal legend if TRUE. Default is FALSE.
arrow.size	size of the arrows. Default is 0.08.
arrow.size.head	size of the arrow head. Default is 0.05.
arrow.lwd	width/thickness of arrows. Default is 2.
arrow.cutoff	cutoff value for plotting arrows. z-values that fall below the arrow.cutoff quantile will not be plotted. Default is 0.9.
arrow.col	Color of arrows. Default is black.
arrow.nlocs.x	Integer number of x- or time-locations where phase arrows should be plotted. Default is 1/2 of the time-locations
arrow.nlocs.y	Integer number of y- or period-locations where phase arrows should be plotted. Default is 1/2 of the period-locations
xlim	the x limits. The default is NULL.
ylim	the y limits. The default is NULL.
xaxt	Add x-axis? The default is s; use n for none.
yaxt	Add y-axis? The default is s; use n for none.
form	format to use to display dates on the x-axis. Default is '%Y' for 4-digit year. See ?Date for other valid formats.
...	other parameters.

### Details

Arrows pointing to the right mean that x and y are in phase.

Arrows pointing to the left mean that x and y are in anti-phase.

Arrows pointing up mean that y leads x by  $\pi/2$ .

Arrows pointing down mean that x leads y by  $\pi/2$ .

### Author(s)

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- Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.
- Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.
- Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.
- Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.

## See Also

[image.plot](#)

## Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Continuous wavelet transform
wt.t1=wt(t1)
## Plot power
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)

## Cross-wavelet transform
xwt.t1t2=xwt(t1, t2)
## Plot cross-wavelet
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(xwt.t1t2, plot.cb=TRUE)

## Example of bias-correction
t1=sin(seq(from=0, to=2*5*pi, length=1000))
t2=sin(seq(from=0, to=2*15*pi, length=1000))
t3=sin(seq(from=0, to=2*40*pi, length=1000))
## This aggregate time series should have the same power at three distinct periods
s=t1+t2+t3
## Compare plots to see bias-effect on CWT: biased power spectrum artificially
## reduces the power of higher-frequency fluctuations.
wt1=wt(cbind(1:1000, s))
par(mfrow=c(1,2))
plot(wt1, type="power.corr.norm", main="Bias-corrected")
plot(wt1, type="power.norm", main="Biased")

## Compare plots to see bias-effect on XWT: biased power spectrum artificially
## reduces the power of higher-frequency fluctuations.
x1=xwt(cbind(1:1000, s), cbind(1:1000, s))
par(mfrow=c(1,2))
plot(x1, type="power.corr.norm", main="Bias-corrected")
plot(x1, type="power.norm", main="Biased")
```

pwtc

*Compute partial wavelet coherence***Description**

Compute partial wavelet coherence between  $y$  and  $x_1$  by partialling out the effect of  $x_2$

**Usage**

```
pwtc (y, x1, x2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
      max.scale = NULL, mother = c("morlet", "paul", "dog"), param = -1,
      lag1 = NULL, sig.level = 0.95, sig.test = 0, nrand = 300, quiet = FALSE)
```

**Arguments**

$y$	time series $y$ in matrix format ( $n$ rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
$x_1$	time series $x_1$ in matrix format ( $n$ rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
$x_2$	time series $x_2$ whose effects should be partialled out in matrix format ( $n$ rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
<code>pad</code>	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
<code>dj</code>	spacing between successive scales. Default is 1/12.
<code>s0</code>	smallest scale of the wavelet. Default is 2*dt.
<code>J1</code>	number of scales - 1.
<code>max.scale</code>	maximum scale. Computed automatically if left unspecified.
<code>mother</code>	type of mother wavelet function to use. Can be set to <code>morlet</code> , <code>dog</code> , or <code>paul</code> . Default is <code>morlet</code> . Significance testing is only available for <code>morlet</code> wavelet.
<code>param</code>	nondimensional parameter specific to the wavelet function.
<code>lag1</code>	vector containing the AR(1) coefficient of each time series.
<code>sig.level</code>	significance level. Default is 0.95.
<code>sig.test</code>	type of significance test. If set to 0, use a regular $\chi^2$ test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
<code>nrand</code>	number of Monte Carlo randomizations. Default is 300.
<code>quiet</code>	Do not display progress bar. Default is FALSE

**Value**

Return a biwavelet object containing:

coi	matrix containing cone of influence
wave	matrix containing the cross-wavelet transform of y and x1
rsq	matrix of partial wavelet coherence between y and x1 (with x2 partialled out)
phase	matrix of phases between y and x1
period	vector of periods
scale	vector of scales
dt	length of a time step
t	vector of times
xaxis	vector of values used to plot xaxis
s0	smallest scale of the wavelet
dj	spacing between successive scales
y.sigma	standard deviation of y
x1.sigma	standard deviation of x1
mother	mother wavelet used
type	type of biwavelet object created (pwtc)
signif	matrix containing sig. level percentiles of wavelet coherence based on the Monte Carlo AR(1) time series

**Note**

The Monte Carlo randomizations can be extremely slow for large datasets. For instance, 1000 randomizations of a dataset consisting of 1000 samples will take ~30 minutes on a 2.66 GHz dual-core Xeon processor.

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

**References**

- Aguiar-Conraria, L., and M. J. Soares. 2013. The Continuous Wavelet Transform: moving beyond uni- and bivariate analysis. *Journal of Economic Surveys* In press.
- Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.
- Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.
- Ng, E. K. W., and J. C. L. Chan. 2012. Geophysical applications of partial wavelet coherence and multiple wavelet coherence. *Journal of Atmospheric and Oceanic Technology* 29:1845-1853.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

### Examples

```
y=cbind(1:100, rnorm(100))
x1=cbind(1:100, rnorm(100))
x2=cbind(1:100, rnorm(100))
## Partial wavelet coherence of y and x1
pwtc.yx1=pwtc(y, x1, x2, nrand=0)
## Partial wavelet coherence of y and x2
pwtc.yx2=pwtc(y, x2, x1, nrand=0)
## Plot partial wavelet coherence and phase difference (arrows)
## Make room to the right for the color bar
par(mfrow=c(2,1), oma=c(4, 0, 0, 1), mar=c(1, 4, 4, 5), mgp = c(1.5, 0.5, 0))
plot(pwtc.yx1, xlab="", plot.cb=TRUE, main="Partial wavelet coherence of y and x1 | x2")
plot(pwtc.yx2, plot.cb=TRUE, main="Partial wavelet coherence of y and x2 | x1")
```

---

smooth.wavelet

*Smooth wavelet in both the time and scale domains*

---

### Description

The time smoothing uses a filter given by the absolute value of the wavelet function at each scale, normalized to have a total weight of unity, which is a Gaussian function for the Morlet wavelet. The scale smoothing is done with a boxcar function of width 0.6, which corresponds to the decorrelation scale of the Morlet wavelet.

### Usage

```
smooth.wavelet (wave, dt, dj, scale)
```

### Arguments

wave	wavelet coefficients
dt	size of time steps
dj	number of octaves per scale
scale	wavelet scales

### Value

Returns the smoothed wavelet.

### Note

This function is used internally for computing wavelet coherence. It is only appropriate for the morlet wavelet.

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)  
Code based on WTC MATLAB package written by Aslak Grinsted.

**References**

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

**Examples**

```
## Not run: smooth.wt1=smooth.wavelet(wave, dt, dj, scale)
```

---

wclust	<i>Compute dissimilarity between multiple wavelet spectra</i>
--------	---

---

**Description**

Compute dissimilarity between multiple wavelet spectra

**Usage**

```
wclust (w.arr)
```

**Arguments**

w.arr	N x p x t array of wavelet spectra where N is the number of wavelet spectra to be compared, p is the number of periods in each wavelet spectrum and t is the number of time steps in each wavelet spectrum.
-------	---

**Value**

Returns a list containing:

diss.mat	square dissimilarity matrix
dist.mat	(lower triangular) distance matrix

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

**References**

Rouyer, T., J. M. Fromentin, F. Menard, B. Cazelles, K. Briand, R. Pianet, B. Planque, and N. C. Stenseth. 2008. Complex interplays among population dynamics, environmental forcing, and exploitation in fisheries. *Proceedings of the National Academy of Sciences* 105:5420-5425.

Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.



**Examples**

```

t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
t3=cbind(1:100, rnorm(100))

## Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
wt.t3=wt(t3)

## Store all wavelet spectra into array
w.arr=array(NA, dim=c(3, NROW(wt.t1$wave), NCOL(wt.t1$wave)))
w.arr[1, , ]=wt.t1$wave
w.arr[2, , ]=wt.t2$wave
w.arr[3, , ]=wt.t3$wave

## Compute dissimilarity and distance matrices
w.arr.dis=wclust(w.arr)
plot(hclust(w.arr.dis$dist.mat, method="ward"), sub="", main="",
      ylab="Dissimilarity", hang=-1)

```

wdist

*Compute dissimilarity between two wavelet spectra***Description**

Compute dissimilarity between two wavelet spectra

**Usage**

```
wdist (wt1, wt2, cutoff = 0.99)
```

**Arguments**

wt1	power, wave or rsq matrix from biwavelet object generated by wt, xwt, or wtc.
wt2	power, wave or rsq matrix from biwavelet object generated by wt, xwt, or wtc.
cutoff	cutoff value used to compute dissimilarity. Only orthogonal axes that contribute more than $1 - \text{cutoff}$ to the total covariance between the two wavelet spectra will be used to compute their dissimilarity. Default is 0.99.

**Value**

Returns wavelet dissimilarity.

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

## References

Rouyer, T., J. M. Fromentin, F. Menard, B. Cazelles, K. Briand, R. Pianet, B. Planque, and N. C. Stenseth. 2008. Complex interplays among population dynamics, environmental forcing, and exploitation in fisheries. *Proceedings of the National Academy of Sciences* 105:5420-5425.

Rouyer, T., J. M. Fromentin, N. C. Stenseth, and B. Cazelles. 2008. Analysing multiple time series and extending significance testing in wavelet analysis. *Marine Ecology Progress Series* 359:11-23.

## Examples

```
t1=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)))
t2=cbind(1:100, sin(seq(from=0, to=10*2*pi, length.out=100)+0.1*pi))
## Compute wavelet spectra
wt.t1=wt(t1)
wt.t2=wt(t2)
## Compute dissimilarity
wdist(wt.t1$wave, wt.t2$wave)
```

---

<code>wt</code>	<i>Compute wavelet transform</i>
-----------------	----------------------------------

---

## Description

Continuous wavelet transform

## Usage

```
wt (d, pad = TRUE, dt = NULL, dj = 1/12, s0 = 2 * dt, J1 = NULL,
    max.scale = NULL, mother = c("morlet", "paul", "dog"),
    param = -1, lag1 = NULL, sig.level = 0.95, sig.test = 0, do.sig=TRUE)
```

## Arguments

<code>d</code>	time series in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
<code>pad</code>	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
<code>dt</code>	length of a time step.
<code>dj</code>	spacing between successive scales. Default is 1/12.
<code>s0</code>	smallest scale of the wavelet. Default is 2*dt
<code>J1</code>	number of scales - 1.
<code>max.scale</code>	maximum scale. Computed automatically if left unspecified.
<code>mother</code>	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet.
<code>param</code>	nondimensional parameter specific to the wavelet function.

lag1	AR(1) coefficient of time series used to test for significant patterns.
sig.level	significance level. Default is 0.95.
sig.test	type of significance test. If set to 0, use a regular $\chi^2$ test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
do.sig	perform significance testing if TRUE. Default is TRUE.

### Value

Returns a `biwavelet` object containing:

coi	matrix containing cone of influence
wave	matrix containing the wavelet transform
power	matrix of power
power.corr	matrix of bias-corrected power using the method described by Liu et al. (2007)
phase	matrix of phases
period	vector of periods
scale	vector of scales
dt	length of a time step
t	vector of times
xaxis	vector of values used to plot xaxis
s0	smallest scale of the wavelet
dj	spacing between successive scales
sigma2	variance of time series
mother	mother wavelet used
type	type of <code>biwavelet</code> object created ( <code>wt</code> )
signif	matrix containing significance levels

### Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gilbert P. Compo.

### References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Liu, Y., X. San Liang, and R. H. Weisberg. 2007. Rectification of the Bias in the Wavelet Power Spectrum. *Journal of Atmospheric and Oceanic Technology* 24:2093-2102.

**Examples**

```
t1=cbind(1:100, rnorm(100))
## Continuous wavelet transform
wt.t1=wt(t1)
## Plot power
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wt.t1, plot.cb=TRUE, plot.phase=FALSE)
```

wt.bases

*Compute wavelet***Description**

Computes the wavelet as a function of Fourier frequency.

**Usage**

```
wt.bases (mother = c("morlet", "paul", "dog"), k, scale, param = -1)
```

**Arguments**

mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet.
k	vector of frequencies at which to calculate the wavelet.
scale	the wavelet scale.
param	nondimensional parameter specific to the wavelet function.

**Value**

Returns a list containing:

daughter	wavelet function
fourier.factor	ratio of fourier period to scale
coi	cone of influence
dof	degrees of freedom for each point in wavelet power

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

**References**

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

**Examples**

```
## Not run: wb=wt.bases(mother, k, scale[a1], param)
```

---

wt.sig *Determine significance of wavelet transform*

---

**Description**

Determine significance of wavelet transform

**Usage**

```
wt.sig (d, dt, scale, sig.test = 0, sig.level = 0.95, dof = 2,
        lag1 = NULL, mother = c("morlet", "paul", "dog"),
        param = -1, sigma2 = NULL)
```

**Arguments**

d	time series in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
dt	length of a time step.
scale	the wavelet scale.
sig.test	type of significance test. If set to 0, use a regular $\chi^2$ test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
sig.level	significance level. Default is 0.95.
dof	degrees of freedom for each point in wavelet power.
lag1	AR(1) coefficient of time series used to test for significant patterns.
mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet.
param	nondimensional parameter specific to the wavelet function.
sigma2	variance of time series

**Value**

Returns a list containing:

signif	vector containing significance level for each scale
signif	vector of red-noise spectrum for each period

**Author(s)**

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on wavelet MATLAB program written by Christopher Torrence and Gibert P. Compo.

## References

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

## Examples

```
## Not run: wt.sig (d, dt, scale, sig.test, sig.level, lag1, dof=-1,
## mother=morlet, sigma2=1)
```

---

wtc *Compute wavelet coherence*

---

## Description

Compute wavelet coherence

## Usage

```
wtc (d1, d2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
     max.scale = NULL, mother = c("morlet", "paul", "dog"), param = -1,
     lag1 = NULL, sig.level = 0.95, sig.test = 0, nrand = 300, quiet = FALSE)
```

## Arguments

d1	time series 1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
d2	time series 2 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
pad	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
dj	spacing between successive scales. Default is 1/12.
s0	smallest scale of the wavelet. Default is 2*dt.
J1	number of scales - 1.
max.scale	maximum scale. Computed automatically if left unspecified.
mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet. Significance testing is only available for morlet wavelet.
param	nondimensional parameter specific to the wavelet function.
lag1	vector containing the AR(1) coefficient of each time series.
sig.level	significance level. Default is 0.95.
sig.test	type of significance test. If set to 0, use a regular $\chi^2$ test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.
nrand	number of Monte Carlo randomizations. Default is 300.
quiet	Do not display progress bar. Default is FALSE

**Value**

Return a `biwavelet` object containing:

<code>coi</code>	matrix containing cone of influence
<code>wave</code>	matrix containing the cross-wavelet transform
<code>wave.corr</code>	matrix containing the bias-corrected cross-wavelet transform using the method described by Veleda et al. (2012)
<code>power</code>	matrix of power
<code>power.corr</code>	matrix of bias-corrected cross-wavelet power using the method described by Veleda et al. (2012)
<code>rsq</code>	matrix of wavelet coherence
<code>phase</code>	matrix of phases
<code>period</code>	vector of periods
<code>scale</code>	vector of scales
<code>dt</code>	length of a time step
<code>t</code>	vector of times
<code>xaxis</code>	vector of values used to plot xaxis
<code>s0</code>	smallest scale of the wavelet
<code>dj</code>	spacing between successive scales
<code>d1.sigma</code>	standard deviation of time series 1
<code>d2.sigma</code>	standard deviation of time series 2
<code>mother</code>	mother wavelet used
<code>type</code>	type of <code>biwavelet</code> object created ( <code>wtc</code> )
<code>signif</code>	matrix containing sig. level percentiles of wavelet coherence based on the Monte Carlo AR(1) time series

**Note**

The Monte Carlo randomizations can be extremely slow for large datasets. For instance, 1000 randomizations of a dataset consisting of 1000 samples will take ~30 minutes on a 2.66 GHz dual-core Xeon processor.

**Author(s)**

Tarik C. Gouhier ([tarik.gouhier@gmail.com](mailto:tarik.gouhier@gmail.com))

Code based on WTC MATLAB package written by Aslak Grinsted.

## References

- Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.
- Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.
- Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.
- Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.
- Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

## Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Wavelet coherence
wtc.t1t2=wtc(t1, t2, nrands=10)
## Plot wavelet coherence and phase difference (arrows)
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(wtc.t1t2, plot.cb=TRUE)
```

---

wtc.sig

*Determine significance of wavelet coherence*

---

## Description

Determine significance of wavelet coherence

## Usage

```
wtc.sig (nrands = 300, lag1, dt, ntimesteps, pad = TRUE, dj = 1/12, s0, J1,
        max.scale=NULL, mother = c("morlet", "paul", "dog"), sig.level = 0.95,
        quiet = FALSE)
```

## Arguments

nrands	number of Monte Carlo randomizations. Default is 300.
lag1	vector containing the AR(1) coefficient of each time series.
dt	length of a time step.
ntimesteps	number of time steps in time series.
pad	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
dj	spacing between successive scales. Default is 1/12.





xwt

*Compute cross-wavelet***Description**

Compute cross-wavelet

**Usage**

```
xwt (d1, d2, pad = TRUE, dj = 1/12, s0 = 2 * dt, J1 = NULL,
     max.scale = NULL, mother = c("morlet", "paul", "dog"),
     param = -1, lag1 = NULL, sig.level = 0.95, sig.test = 0)
```

**Arguments**

d1	time series 1 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
d2	time series 2 in matrix format (n rows x 2 columns). The first column should contain the time steps and the second column should contain the values.
pad	pad the values will with zeros to increase the speed of the transform. Default is TRUE.
dj	spacing between successive scales. Default is 1/12.
s0	smallest scale of the wavelet. Default is 2*dt
J1	number of scales - 1.
max.scale	maximum scale. Computed automatically if left unspecified.
mother	type of mother wavelet function to use. Can be set to morlet, dog, or paul. Default is morlet. Significance testing is only available for morlet wavelet.
param	nondimensional parameter specific to the wavelet function.
lag1	vector containing the AR(1) coefficient of each time series.
sig.level	significance level. Default is 0.95.
sig.test	type of significance test. If set to 0, use a regular $\chi^2$ test. If set to 1, then perform a time-average test. If set to 2, then do a scale-average test.

**Value**

Returns a biwavelet object containing:

coi	matrix containing cone of influence
wave	matrix containing the cross-wavelet transform
wave.corr	matrix containing the bias-corrected cross-wavelet transform using the method described by Veleda et al. (2012)
power	matrix of power

power.corr	matrix of bias-corrected cross-wavelet power using the method described by Veleda et al. (2012)
phase	matrix of phases
period	vector of periods
scale	vector of scales
dt	length of a time step
t	vector of times
xaxis	vector of values used to plot xaxis
s0	smallest scale of the wavelet
dj	spacing between successive scales
d1.sigma	standard deviation of time series 1
d2.sigma	standard deviation of time series 2
mother	mother wavelet used
type	type of biwavelet object created (xwt)
signif	matrix containing significance levels

### Author(s)

Tarik C. Gouhier (tarik.gouhier@gmail.com)

Code based on WTC MATLAB package written by Aslak Grinsted.

### References

Cazelles, B., M. Chavez, D. Berteaux, F. Menard, J. O. Vik, S. Jenouvrier, and N. C. Stenseth. 2008. Wavelet analysis of ecological time series. *Oecologia* 156:287-304.

Grinsted, A., J. C. Moore, and S. Jevrejeva. 2004. Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear Processes in Geophysics* 11:561-566.

Torrence, C., and G. P. Compo. 1998. A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society* 79:61-78.

Torrence, C., and P. J. Webster. 1998. The annual cycle of persistence in the El Nino/Southern Oscillation. *Quarterly Journal of the Royal Meteorological Society* 124:1985-2004.

Veleda, D., R. Montagne, and M. Araujo. 2012. Cross-Wavelet Bias Corrected by Normalizing Scales. *Journal of Atmospheric and Oceanic Technology* 29:1401-1408.

### Examples

```
t1=cbind(1:100, rnorm(100))
t2=cbind(1:100, rnorm(100))
## Cross-wavelet
xwt.t1t2=xwt(t1, t2)
## Plot cross-wavelet and phase difference (arrows)
plot(xwt.t1t2, plot.cb=TRUE, plot.phase=TRUE)
## Real data
data(enviro.data)
```

```
## Cross-wavelet of MEI and NPGO
xwt.mei.npgo=xwt(subset(enviro.data, select=c("date", "mei")),
  subset(enviro.data, select=c("date", "npgo")))
## Make room to the right for the color bar
par(oma=c(0, 0, 0, 1), mar=c(5, 4, 4, 5) + 0.1)
plot(xwt.mei.npgo, plot.cb=TRUE, plot.phase=TRUE)
```

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