

Package ‘agop’

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Title Aggregation Operators and Preordered Sets

Description Tools supporting multi-criteria decision making, including variable number of criteria, by means of aggregation operators and preordered sets. Possible applications include, but are not limited to, scientometrics and bibliometrics.

URL <http://agop.rexamine.com/>

BugReports <http://github.com/Rexamine/agop/issues>

ByteCompile TRUE

Type Package

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agop-package

Aggregation Operators Package for R

Description

“The process of combining several numerical values into a single representative one is called aggregation, and the numerical function performing this process is called aggregation function. This simple definition demonstrates the size of the field of application of aggregation: applied mathematics (e.g. probability, statistics, decision theory), computer science (e.g. artificial intelligence, operation research), as well as many applied fields (economics and finance, pattern recognition and image processing, data fusion, multicriteria decision making, automated reasoning etc.). Although history of aggregation is probably as old as mathematics (think of the arithmetic mean), its existence has reminded underground till only recent (...).” (Grabisch et al, 2009, p. xiii)

Details

agop is an open source (LGPL 3) package for R, to which anyone can contribute. It started as a fork of the **CITAN** package (Gagolewski, 2011).

For more information refer to the Package Vignette. Its most recent version is available at <http://github.com/Rexamine/agop/raw/master/inst/doc/agop-Tutorial.pdf>.

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closure_total_fair *Total Closure of Adjacency Matrix [Fair Totalization]*

Description

Fair totalization: for each pair (x,y) s.t. not xRy and not xRy let from now on xRy and yRx

Usage

closure_total_fair(B)

Arguments

B object of class igraph or a square 0-1 matrix of class Matrix or matrix

Details

If you want a total preorder, call [closure_transitive](#).

Value

object of class Matrix

See Also

Other binary_relations: [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

closure_transitive *Transitive Closure of Adjacency Matrix*

Description

This may be slow for large graphs.

Usage

```
closure_transitive(B)
```

Arguments

B object of class igraph or a square 0-1 matrix of class Matrix or matrix

Value

object of class Matrix

See Also

Other binary_relations: [closure_total_fair](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

de_transitive *De-transitivitize Graph*

Description

Useful for draving Hasse diagrams.

Usage

```
de_transitive(B)
```

Arguments

B object of class igraph or a square 0-1 matrix of class Matrix or matrix

Value

object of class Matrix

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

get_equivalence_classes

Get All Equivalence Classes of a Total Binary Relation

Description

Note that we assume that B is total, reflexive and transitive.

Usage

```
get_equivalence_classes(B)
```

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Value

list of integer vectors; each list element defines an equivalence class by listing vertices' numbers; each vector is ordered by the outdegrees of their nodes (they are the same in each class)

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

get_incomparable_pairs

Get Incomparable Pairs in an Adjacency Matrix

Description

A pair (x,y) is incomparable iff not xRy and not xRy

Usage

```
get_incomparable_pairs(B)
```

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Details

See also [get_independent_sets](#) of how to generate all maximal independent sets.

Value

integer matrix with two columns (indices of incomparable elements, not that these are pairs, and not sets: you'll get (i,j) and (j,i))

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

get_independent_sets *Get All Maximal Independent Sets*

Description

The function generates vectors of indices $S_j = \{i_1, \dots, i_{k_j}\}$ such that all pairs from S_j are incomparable (A pair (i,i') is incomparable iff not iRi' and not $i'Ri$, see also [get_incomparable_pairs](#)).

Usage

```
get_independent_sets(B)
```

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Details

Note that we assume that B is transitive. Loops are not taken into account at all.

Value

list of integer vectors; each list element defines an independent set of vertices numbers

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

index_g

*Egghe's g-index***Description**

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j \geq 0$ for $i \leq j$, the *g-index* (Egghe, 2006) for x is defined as

$$G(x) = \max\{i = 1, \dots, n : \sum_{j=1}^i x_j \geq i^2\}$$

if $n \geq 1$ and $x_1 \geq 1$, or $G(x) = 0$ otherwise.

Usage

index_g(x)

index.g(x) # same as index_g(x), deprecated alias

index_g_zi(x)

Arguments

x a non-negative numeric vector

Details

index.g is a (deprecated) alias for index_g.

Note that index_g is not a zero-insensitive impact function, see Examples section. index_g_zi is its zero-sensitive variant: it assumes that the aggregated vector is padded with zeros.

The h-index is the same as the discrete Sugeno integral of x w.r.t. the counting measure (cf. Torra, Narukawa, 2008).

If non-increasingly sorted vector is given, the function is $O(n)$.

For historical reasons, this function is also available via its alias, index.h [but its usage is deprecated].

Value

a single numeric value

References

Egghe L., Theory and practise of the g-index, *Scientometrics* 69(1), 131-152, 2006.

Torra V., Narukawa Y., The h-index and the number of citations: Two fuzzy integrals. *IEEE Transactions on Fuzzy Systems* 16(3), 2008, 795-797.

See Also

Other impact_functions: [index.h](#), [index_h](#); [index.lp](#), [index_lp](#); [index.rp](#), [index_rp](#); [index_maxprod](#); [index_w](#)

Examples

```
sapply(list(c(9), c(9,0), c(9,0,0), c(9,0,0,0)), index_g) # not a zero-sensitive agop
```

index_h	<i>Hirsch's h-index</i>
---------	-------------------------

Description

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j \geq 0$ for $i \leq j$, the *h-index* (Hirsch, 2005) for x is defined as

$$H(x) = \max\{i = 1, \dots, n : x_i \geq i\}$$

if $n \geq 1$ and $x_1 \geq 1$, or $H(x) = 0$ otherwise.

Usage

```
index_h(x)
```

```
index.h(x) # same as index_h(x), deprecated alias
```

Arguments

`x` a non-negative numeric vector

Details

If non-increasingly sorted vector is given, the function is $O(n)$.

For historical reasons, this function is also available via its alias, `index.h` [but its usage is deprecated].

See [index_rp](#) and [owmax](#) for natural generalizations.

Value

a single numeric value

References

Hirsch J.E., An index to quantify individual's scientific research output, Proceedings of the National Academy of Sciences 102(46), 16569-16572, 2005.

See Also

Other impact_functions: [index.g](#), [index_g](#), [index_g_zi](#); [index.lp](#), [index_lp](#); [index.rp](#), [index_rp](#); [index_maxprod](#); [index_w](#)

Examples

```
authors <- list( # a list of numeric sequences
               # (e.g. citation counts of the articles
               # written by some authors)
  "A" =c(23,21,4,2,1,0,0),
  "B" =c(11,5,4,4,3,2,2,2,2,1,1,1,0,0,0,0),
  "C" =c(53,43,32,23,14,13,12,8,4,3,2,1,0)
)
index_h(authors$A)
sapply(authors, index_h)
```

 index_lp

The l_p -index

Description

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j$ for $i \leq j$, the l_p -index for $p = \infty$ equals to

$$l_p(x) = \arg \max_{(i, x_i), i=1, \dots, n} \{ix_i\}$$

if $n \geq 1$, or $l_\infty(x) = 0$ otherwise. Note that if $(i, x_i) = l_\infty(x)$, then

$$MAXPROD(x) = \text{prod}(l_\infty(x)) = ix_i,$$

where *MAXPROD* is the index proposed in (Kosmulski, 2007), see [index_maxprod](#).

For the definition of the l_p -index for $p < \infty$ we refer to (Gagolewski, Grzegorzewski, 2009a).

Usage

```
index_lp(x, p = Inf, projection = prod)
```

```
index.lp(x, p = Inf, projection = prod) # deprecated alias
```

Arguments

x a non-negative numeric vector
p index order, $p \in [1, \infty]$; defaults ∞ (Inf).
projection function

Details

The l_p -index, by definition, is not an impact function, as it produces 2 numeric values. Thus, it should be projected to one dimension. However, you may set projection to `identity` to obtain the 2-dimensional index

If non-increasingly sorted vector is given, the function is $O(n)$.

For historical reasons, this function is also available via its alias, `index.lp` [but its usage is deprecated].

Value

result of `projection(c(i, xi))`

References

Gagolewski M., Grzegorzewski P., A geometric approach to the construction of scientific impact indices, *Scientometrics*, 81(3), 2009a, pp. 617-634.

Gagolewski M., Debski M., Nowakiewicz M., Efficient algorithms for computing "geometric" scientific impact indices, Research Report of Systems Research Institute, Polish Academy of Sciences RB/1/2009, 2009b.

Kosmulski M., MAXPROD - A new index for assessment of the scientific output of an individual, and a comparison with the h-index, *Cybermetrics*, 11(1), 2007.

See Also

Other impact_functions: [index.g](#), [index_g](#), [index_g_zi](#); [index.h](#), [index_h](#); [index.rp](#), [index_rp](#); [index_maxprod](#); [index_w](#)

Examples

```
x <- runif(100, 0, 100)
index.lp(x, Inf, identity) # two-dimensional value, can not be used
                          # directly in the analysis
index.lp(x, Inf, prod)    # the MAXPROD-index (one-dimensional) [default]
```

index_maxprod

Kosmulski's MAXPROD-index

Description

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j \geq 0$ for $i \leq j$, the *MAXPROD-index* (Kosmulski, 2007) for x is defined as

$$MAXPROD(x) = \max\{ix_i : i = 1, \dots, n\}$$

Usage

`index_maxprod(x)`

Arguments

`x` a non-negative numeric vector

Details

If non-increasingly sorted vector is given, the function is O(n).

MAXPROD index is the same as the discrete Shilkret integral of `x` w.r.t. the counting measure.

See [index_lp](#) for a natural generalization.

Value

a single numeric value

References

Kosmulski M., MAXPROD - A new index for assessment of the scientific output of an individual, and a comparison with the h-index, *Cybermetrics* 11(1), 2007.

See Also

Other impact_functions: [index.g](#), [index_g](#), [index_g_zi](#); [index.h](#), [index_h](#); [index.lp](#), [index_lp](#); [index rp](#), [index_rp](#); [index_w](#)

index_rp

The r_p-index

Description

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j$ for $i \leq j$, the r_p -index for $p = \infty$ equals to

$$r_p(x) = \max_{i=1, \dots, n} \{\min\{i, x_i\}\}$$

if $n \geq 1$, or $r_\infty(x) = 0$ otherwise. That is, it is equivalent to a particular OWMMax operator, see [owmax](#).

For the definition of the r_p -index for $p < \infty$ we refer to (Gagolewski, Grzegorzewski, 2009).

Usage

```
index_rp(x, p = Inf)
```

```
index.rp(x, p = Inf) # same as index_rp(x, p), deprecated alias
```

Arguments

`x` a non-negative numeric vector

`p` index order, $p \in [1, \infty]$; defaults ∞ (Inf).

Details

Note that if x_1, \dots, x_n are integers, then

$$r_\infty(x) = H(x),$$

where H is the h -index (Hirsch, 2005) and

$$r_1(x) = W(x),$$

where W is the w -index (Woeginger, 2008), see [index_h](#) and [index_w](#).

If non-increasingly sorted vector is given, the function is $O(n)$.

For historical reasons, this function is also available via its alias, `index.rp` [but its usage is deprecated].

Value

a single numeric value

References

Gagolewski M., Grzegorzewski P., A geometric approach to the construction of scientific impact indices, *Scientometrics*, 81(3), 2009, pp. 617-634.

Hirsch J.E., An index to quantify individual's scientific research output, *Proceedings of the National Academy of Sciences* 102(46), 16569-16572, 2005.

Woeginger G.J., An axiomatic characterization of the Hirsch-index, *Mathematical Social Sciences*, 56(2), 224-232, 2008.

See Also

Other impact_functions: [index.g](#), [index_g](#), [index_g_zi](#); [index.h](#), [index_h](#); [index.lp](#), [index_lp](#); [index_maxprod](#); [index_w](#)

Examples

```
x <- runif(100, 0, 100);
index.rp(x);           # the r_oo-index
floor(index.rp(x));   # the h-index
index.rp(floor(x), 1); # the w-index
```

index_w

Woeginger's w-index

Description

Given a sequence of n non-negative numbers $x = (x_1, \dots, x_n)$, where $x_i \geq x_j \geq 0$ for $i \leq j$, the w -index (Woeginger, 2008) for x is defined as

$$W(x) = \max\{i = 1, \dots, n : x_j \geq i - j + 1, \forall j = 1, \dots, i\}$$

Usage

```
index_w(x)
```

Arguments

x a non-negative numeric vector

Details

If non-increasingly sorted vector is given, the function is $O(n)$.

See [index_rp](#) for a natural generalization.

Value

a single numeric value

References

Woeginger G. J., An axiomatic characterization of the Hirsch-index. *Mathematical Social Sciences* 56(2), 2008, 224-232.

See Also

Other impact_functions: [index.g](#), [index_g](#), [index_g_zi](#); [index.h](#), [index_h](#); [index.lp](#), [index_lp](#); [index_rp](#), [index_rp](#); [index_maxprod](#)

is_reflexive

Check if Given Adjacency Matrix is Reflexive

Description

A binary relation R is reflexive, iff for all x we have xRx . The function just checks whether all elements on the diagonal of B are non-zeros.

Usage

```
is_reflexive(B)
```

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Value

single logical value

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_total](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

is_total	<i>Check if Given Adjacency Matrix is Total</i>
----------	---

Description

A binary relation R is total, iff for all x, y we have xRy or yRx .

Usage

is_total(B)

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Value

single logical value

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_transitive](#); [pord_weakdom](#); [rel_graph](#)

is_transitive	<i>Check if Given Adjacency Matrix is Transitive</i>
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Description

A binary relation R is transitive, iff for all x, y, z we have xRy and $yRz \Rightarrow xRz$

Usage

is_transitive(B)

Arguments

B object of class `igraph` or a square 0-1 matrix of class `Matrix` or `matrix`

Value

single logical value

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [pord_weakdom](#); [rel_graph](#)

 owa

WAM and OWA Operators

Description

Computes the Wegtled Arithmetic Mean or the Ordered Weighted Averaging aggregation operator.

Usage

```
owa(x, w = rep(1/length(x), length(x)))
```

```
wam(x, w = rep(1/length(x), length(x)))
```

Arguments

x numeric vector to be aggregated
w numeric vector of the same length as x, with elements in $[0, 1]$, and such that $\sum_i w_i = 1$; weights

Details

The OWA operator is given by

$$\text{OWA}_w(\mathbf{x}) = \sum_{i=1}^n w_i x_{\{i\}}$$

where $x_{\{i\}}$ denotes the i -th greatest value in \mathbf{x} .

The WAM operator is given by

$$\text{WAM}_w(\mathbf{x}) = \sum_{i=1}^n w_i x_i$$

If the elements of w does not sum up to 1, then they are normalized and a warning is generated.

Both functions return the ordinary arithmetic mean by default. Special cases of OWA include the trimmed mean (cf. [mean](#)) and winsorized mean.

There is a strong connection between the OWA operators and the Choquet integrals.

Value

single numeric value

References

Yager R.R., On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Transactions on Systems, Man, and Cybernetics 18(1), 1988, pp. 183-190.

See Also

Other aggregation_operators: [owmax](#), [owmin](#), [wmax](#), [wmin](#)

owmax	<i>WMax, WMin, OWMax, and OWMin Operators</i>
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Description

Computes the (Ordered) Weighted Maximum/Minimum.

Usage

`owmax(x, w = rep(Inf, length(x)))`

`owmin(x, w = rep(-Inf, length(x)))`

`wmax(x, w = rep(Inf, length(x)))`

`wmin(x, w = rep(-Inf, length(x)))`

Arguments

`x` numeric vector to be aggregated
`w` numeric vector of the same length as `x`; weights

Details

The OWMax operator is given by

$$\text{OWMax}_w(\mathbf{x}) = \bigvee_{i=1}^n w_i \wedge x_{\{i\}}$$

where $x_{\{i\}}$ denotes the i -th greatest value in \mathbf{x} .

The OWMin operator is given by

$$\text{OWMin}_w(\mathbf{x}) = \bigwedge_{i=1}^n w_i \vee x_{\{i\}}$$

The WMax operator is given by

$$\text{WMax}_w(\mathbf{x}) = \bigvee_{i=1}^n w_i \wedge x_i$$

The WMin operator is given by

$$\text{WMin}_w(x) = \bigwedge_{i=1}^n w_i \vee x_i$$

OWMax and WMax return the greatest value in x by default, and OWMin and WMin - the smallest value in x .

Note that e.g. in the case of OWMax operator the aggregation w.r.t. w gives the same result as that of w.r.t. $\text{sort}(w)$. Moreover, classically, it is assumed that if we aggregate vectors with elements in $[a, b]$, then the largest weight should be equal to b .

There is a strong connection between the OWMax/OWMin operators and the Sugeno integrals. Additionally, it may be shown that the OWMax and OWMin classes are equivalent.

Moreover, [index_h](#) for integer data is a particular OWMax operator.

Value

single numeric value

References

Dubois D., Prade H., Testemale C., Weighted fuzzy pattern matching, Fuzzy Sets and Systems 28, 1988, pp. 313-331.

See Also

Other aggregation_operators: [owa](#), [wam](#)

pareto2_estimate_mle *Parameter Estimation in the Pareto-II Distribution (MLE)*

Description

Finds the maximum likelihood estimator of the type II Pareto distribution's shape parameter k and, if not given explicitly, scale parameter s .

Usage

```
pareto2_estimate_mle(x, s = NA_real_, smin = 1e-04, smax = 20,
  tol = .Machine$double.eps^0.25)
```

Arguments

<code>x</code>	a non-negative numeric vector
<code>s</code>	a-priori known scale parameter, $s > 0$ or NA if unknown (default)
<code>smin</code>	lower bound for the scale parameter to look for
<code>smax</code>	upper bound for the scale parameter to look for
<code>tol</code>	the desired accuracy (convergence tolerance)

Details

Note that if s is not given, then the maximum of the likelihood function may not exist for some input vectors. This estimator may have large mean squared error. Consider using [pareto2_estimate_mmse](#). For known s , the estimator is unbiased.

Value

a numeric vector with the following named components:

- k - estimated parameter of shape
- s - estimated (or known, see the s argument) parameter of scale

or $c(\text{NA}, \text{NA})$ if the maximum of the likelihood function could not be found.

See Also

Other Pareto2: [dpareto2](#), [ppareto2](#), [qpareto2](#), [rpareto2](#); [pareto2_estimate_mmse](#); [pareto2_test_f](#)

`pareto2_estimate_mmse` *Parameter Estimation in the Pareto-II Distribution (MMSE)*

Description

Finds the MMS estimator of the type II Pareto distribution parameters using the Bayesian method (and the R code) developed by Zhang and Stevens (2009).

Usage

```
pareto2_estimate_mmse(x)
```

Arguments

x a non-negative numeric vector

Value

a numeric vector with the following named components:

- k - estimated parameter of shape,
- s - estimated parameter of scale.

References

Zhang J., Stevens M.A., A New and Efficient Estimation Method for the Generalized Pareto Distribution, *Technometrics* 51(3), 2009, 316-325.

See Also

Other Pareto2: [dpareto2](#), [ppareto2](#), [qpareto2](#), [rpareto2](#); [pareto2_estimate_mle](#); [pareto2_test_f](#)

pareto2_test_f	<i>Two-Sample F-test For Equality of Shape Parameters for Type II-Pareto Distributions</i>
----------------	--

Description

Performs F-test for equality of shape parameters of two samples from the Pareto type-II distributions with known and equal scale parameters, $s > 0$.

Usage

```
pareto2_test_f(x, y, s, alternative = c("two.sided", "less", "greater"),
  significance = NULL)
```

Arguments

x	a non-negative numeric vector
y	a non-negative numeric vector
s	the known scale parameter, $s > 0$
alternative	indicates the alternative hypothesis and must be one of "two.sided" (default), "less", or "greater"
significance	significance level, $0 < \text{significance} < 1$ or NULL. See the Value section for details

Details

Given two samples (X_1, \dots, X_n) i.i.d. $P2(k_x, s)$ and (Y_1, \dots, Y_m) i.i.d. $P2(k_y, s)$ this test verifies the null hypothesis $H_0 : k_x = k_y$ against two-sided or one-sided alternatives, depending on the value of alternative. It bases on test statistic $T(X, Y) = \frac{n \sum_{i=1}^m \log(1+Y_i/m)}{m \sum_{i=1}^n \log(1+X_i/n)}$ which, under H_0 , has the Snedecor's F distribution with $(2m, 2n)$ degrees of freedom.

Note that for $k_x < k_y$, then X dominates Y stochastically.

Value

If significance is not NULL, then the list of class `htest` with the following components is passed as a result:

- `statistic` - the value of the test statistic.
- `result` - either FALSE (accept null hypothesis) or TRUE (reject).
- `alternative` - a character string describing the alternative hypothesis.
- `method` - a character string indicating what type of test was performed.
- `data.name` - a character string giving the name(s) of the data.

Otherwise, the list of class `htest` with the following components is passed as a result:

- statistic the value of the test statistic.
- p.value the p-value of the test.
- alternative a character string describing the alternative hypothesis.
- method a character string indicating what type of test was performed.
- data.name a character string giving the name(s) of the data.

See Also

Other Pareto2: [dpareto2](#), [ppareto2](#), [qpareto2](#), [rpareto2](#); [pareto2_estimate_mle](#); [pareto2_estimate_mmse](#)

plot_producer

Draws a Graphical Representation of a Given Vector

Description

Draws a step function that represents given numeric vector with elements in $[0, \infty]$.

Usage

```
plot_producer(x, type = c("left.continuous", "right.continuous", "curve"),
  extend = FALSE, add = FALSE, pch = 1, col = 1, lty = 1, lwd = 1,
  cex = 1, col.steps = col, lty.steps = 2, lwd.steps = 1, xlab = "",
  ylab = "", main = "", xmarg = 10, xlim = c(0, length(x) * 1.2),
  ylim = c(0, max(x)), ...)
```

Arguments

x	non-negative numeric vector
type	character; type of the graphical 'left.continuous' (the default) or 'right.continuous' for step functions and 'curve' for a continuous step curve
extend	logical; should the plot be extended infinitely to the right? Defaults to FALSE
add	logical; indicates whether to start a new plot, FALSE by default
pch,col,lty,lwd,cex,xmarg	graphical parameters
col.steps,lty.steps,lwd.steps	graphical parameters, used only for type of 'left.continuous' and 'right.continuous' only
ylim,xlim,xlab,ylab,main,...	additional graphical parameters, see plot.default

Details

In **agop**, a given vector $x = (x_1, \dots, x_n)$ can be represented by a step function defined for $0 \leq y < n$ and given by:

$$\pi(y) = x_{(n-\lfloor y+1 \rfloor+1)}$$

(for `type == 'right.continuous'`) or for $0 < y \leq n$

$$\pi(y) = x_{(n-\lfloor y \rfloor+1)}$$

(for `type == 'left.continuous'`, the default) or by a curve joining the points $(0, x_{(n)})$, $(1, x_{(n)})$, $(1, x_{(n-1)})$, $(2, x_{(n-1)})$, ..., $(n, x_{(1)})$. Here, $x_{(i)}$ denotes the i -th smallest value in x .

In bibliometrics, a step function of one of the two above-presented types is called a citation function.

For historical reasons, this function is also available via its alias, `plot.citfun` [but its usage is deprecated].

Value

nothing interesting

Examples

```
john_s <- c(11,5,4,4,3,2,2,2,2,2,1,1,1,0,0,0,0)
plot_producer(john_s, main="Smith, John", col="red")
```

pord_weakdom

Weak Dominance Relation (Preorder)

Description

$x \leq y$ iff $nx \leq ny$ AND for all $i = 1, \dots, n$ $x_{(n-i+1)} \leq y_{(n-i+1)}$, where $nx = \text{length}(x)$ and $ny = \text{length}(y)$.

Usage

```
pord_weakdom(x, y)
```

Arguments

<code>x</code>	numeric vector
<code>y</code>	numeric vector

Details

This function accepts only vectors with nonnegative elements.

Value

single logical value; whether $x \leq y$

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [rel_graph](#)

rel_graph

Create Adjacency Matrix of Given Binary Relation

Description

Note that adjacency matrix can also be conceived as a directed graph (DAG). $ret[i,j]$ iff $i \leq j$

Usage

```
rel_graph(x, pord, ...)
```

Arguments

x	list with elements to compare, preferably named
pord	function with 2 arguments, returning boolean value
...	additional arguments passed to pord

Value

square 0-1 Matrix (of class Matrix)

See Also

Other binary_relations: [closure_total_fair](#); [closure_transitive](#); [de_transitive](#); [get_equivalence_classes](#); [get_incomparable_pairs](#); [get_independent_sets](#); [is_reflexive](#); [is_total](#); [is_transitive](#); [pord_weakdom](#)

rpareto2

Pareto Type-II (Lomax) Distribution

Description

Density, cumulative distribution function, quantile function, and random generation for the Pareto Type-II (Lomax) distribution with shape parameter $k > 0$ and scale parameter $s > 0$.

Usage

```
rpareto2(n, k = 1, s = 1)
ppareto2(q, k = 1, s = 1, lower.tail = TRUE)
qpareto2(p, k = 1, s = 1, lower.tail = TRUE)
dpareto2(x, k = 1, s = 1)
```

Arguments

<code>x, q</code>	vector of quantiles
<code>p</code>	vector of probabilities
<code>n</code>	integer; number of observations
<code>k</code>	vector of shape parameters, $k > 0$
<code>s</code>	vector of scale parameters, $s > 0$
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P(X \leq x)$, and $P(X > x)$ otherwise

Details

If $X \sim P2(k, s)$, then $\text{supp } X = [0, \infty)$. The c.d.f. for $x \geq 0$ is given by

$$F(x) = 1 - s^k / (s + x)^k$$

and the density by

$$f(x) = ks^k / (s + x)^{k+1}.$$

Value

numeric vector; `dpareto2` gives the density, `ppareto2` gives the cumulative distribution function, `qpareto2` calculates the quantile function, and `rpareto2` generates random deviates.

See Also

Other Pareto2: [pareto2_estimate_mle](#); [pareto2_estimate_mmse](#); [pareto2_test_f](#)

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